Exceptional field theory and gauged supergravity

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Outline

1. References
2. 11-d SUGRA, U-dualities and gauged SUGRAs
3. Exceptional field theory
4. IIB theory
5. A geometry: flux formulation
6. $\mathrm{SO}(p, q)$ gaugings
7. Summary
8. Further work

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Talk based on: Berman, Blair, EM, Perry arXiv:1306.6727, Blair, EM, Park arXiv:1311.5109, Blair, EM arXiv:1412.0635 and EM, Samtleben *to appear*.

- EFT / DFT rich subject with long history. Complete references impossible here.
- Built on pioneering work of Tseytlin '90, Siegel '90, Duff '91. Revived by Zwiebach, Hull, Hohm '09, '10 ...
- Uses tools of generalised geometry of Hitchin, Gualtieri. Developed further by Hull, Minasian, Waldram and many others.
- Reviews of DFT / EFT: Hohm, Lüst, Zwiebach (1309.2977), Aldazabal et al (1305.1907), Berman & Thompson (1306.2643).
- Much literature on gauged SUGRA from DFT / EFT: Aldazabal, Andriot, Baron, Berman, Betz, Blumenhagen, Blair, Dall'Agata, Deser, Dibitetto, EM, Fuchs, Geissbuhler, Godazgar^2, Graña, Hassler, Hohm, Kounnas, Larfors, Lüst, Marques, Massai, Musaev, Nicolai, Nunez, Park, Patalong, Plauschinn, Perry, Rennecke, Samtleben, Sun etc.

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11-d SUGRA and U-dualities

- 11-D SUGRA is LEEA of M-theory.
- M-theory has U-duality symmetry: combination of T- and S-duality of string theory.
- We see U-duality in 11-d theory when dimensionally reducing. n commuting killing vectors $\Rightarrow E_n$ symmetry.
- Scalar potential invariant under global E_n and local H_n symmetry.
- Scalars parameterise the coset space E_n/H_n.

n	E_n	H_n
3	$\mathrm{SL}(2) \times \mathrm{SL}(3)$	$\mathrm{SO}(2) \times \mathrm{SO}(3)$
4	$\mathrm{SL}(5)$	$\mathrm{SO}(5)$
5	$\mathrm{SO}(5,5)$	$\mathrm{SO}(5) \times \mathrm{SO}(5)$
6	E_6	$\mathrm{USp}(8)$
7	E_7	$\mathrm{SU}(8)$
8	E_8	$\mathrm{SO}(16)$

Twisted compactification

- Toroidal compactification: flat scalar potential ⇒ phenomenologically unimportant. Role of U-duality clear.
- Twisted (Scherk-Schwarz) compactifications: better!
- There is dependence on internal coordinates.
- Dependence is contained through a "twist" matrix: element of global symmetry group (here E_n).
- Twist can be a U-duality transformation ⇒ U-fold ("non-geometry").
- Non-trivial scalar potential ⇒ moduli stabilisation? cosmological constant?
- Leads to gauged SUGRA. A subgroup of E_n becomes gauged.
- Effective theory still controlled by E_n.

Gauged SUGRA

- Gauged SUGRA classified using embedding tensor Θ_M.
- Θ_M defines which subgroup of E_n is gauged.
- Linear & quadratic constraints on $\Theta_M \Longleftrightarrow$ consistent gauged maximal SUGRA.
- Linear & quadratic constraints are group-theoretic – controlled by E_n.
- Gauged SUGRA important for phenomenology and holography, e.g.
  - Maximal SUSY-preserving deformations of ungauged SUGRA.
  - Fluctuations around $AdS_5 \times S^5$ are governed by gauged SUGRA.
- Focus on maximal gauged SUGRA: a lot of results from just group theory.

Higher dimensional origin of maximal gauged SUGRA

Higher-dimensional origin not always clear!

- Twisted compactification ⇒ Θ_M
- Θ_M ⇒ twisted compactification????

Higher dimensional origin of maximal gauged SUGRA

Higher-dimensional origin not always clear!

- Twisted compactification $\Rightarrow \Theta_M$
- $\Theta_M \Rightarrow$ twisted compactification????

General study of gauged SUGRA hard, even for $d = 7$:

- Quadratic constraint difficult to solve in general.
- Consistency of compactification, even on S^4, S^5, S^7, difficult to prove: deWit, Nicolai '87, Nastase, Vaman, Nieuwenhuizen '00, Cvetic, Lu, Pope '00, deWit, Nicolai '13, …

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EFT can help!

- U-duality E_n plays a prominent role.
- Non-geometric compactifications appear naturally.
- Θ_M is the torsion of EFT.

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- Non-geometric compactifications appear naturally.
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Approach: Use twisted reductions of EFT to obtain new *automatically* consistent compactifications.

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Exceptional field theory

- Consider the bosonic $d = 7$ case ($n = 4$) for simplicity.
- No reduction yet, just a $7 + 4$ split.

$$ds^2 = e^{\phi} ds_7{}^2 + ds_4{}^2 \,. \tag{1}$$

- Focus on scalar potential. Full theory (for other groups): H&S 3×'13, '14. Fermions: Godazgar^2, Hohm, Nicolai, Samtleben '14.
- U-duality mixes bosonic fields: $g_{\alpha\beta}$, $C_{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma\rho} V^{\rho}$, ϕ, $\alpha, \beta = 1, \ldots 4$.
- $\Rightarrow$ combine in $\mathrm{SL}(5)$ tensor, generalised metric: (Hull '07, Berman, Godazgar, Perry '11)

$$m_{ab} = e^{-\phi/2} \begin{pmatrix} \frac{g_{\alpha\beta}}{\sqrt{|g|}} & V_{\alpha} \\ V_{\beta} & \sqrt{g}\left(1 + V^2\right) \end{pmatrix} \,, \quad a, b = 1, \ldots 5 \,. \tag{2}$$

- U-duality mixes 4 momenta and 6 wrapping modes of M2: $p_{\alpha}, w^{[\alpha\beta]}$.
- $\Rightarrow$ introduce winding coords. $y_{\alpha\beta}$ & combine

$$X^{ab} = \begin{cases} X^{\alpha 5} = x^{\alpha} \,, & \text{spacetime coords} \\ X^{\alpha\beta} = \frac{1}{2} \eta^{\alpha\beta\gamma\rho} y_{\gamma\rho} \,, & \eta^{\alpha\beta\gamma\rho} = \sqrt{|g|} \epsilon^{\alpha\beta\gamma\rho} = \pm 1 \,. \end{cases} \tag{3}$$

Generalised diffeomorphisms

- Local symmetries are diffeomorphisms (ξ^α) and gauge transformations ($\lambda_{\alpha\beta}$):

$$
\begin{aligned}
\delta g_{\alpha\beta} &= \mathcal{L}_\xi g_{\alpha\beta} = \xi^\gamma \partial_\gamma g_{\alpha\beta} + g_{\alpha\gamma}\partial_\beta \xi^\gamma + g_{\gamma\beta}\partial_\alpha \xi^\gamma \,, \\
\delta C_{\alpha\beta\gamma} &= \mathcal{L}_\xi C_{\alpha\beta\gamma} - 3\partial_{[\alpha}\lambda_{\beta\gamma]} = \xi^\rho \partial_\rho C_{\alpha\beta\gamma} + 3 C_{\rho[\beta\gamma}\partial_{\alpha]}\xi^\rho - 3\partial_{[\alpha}\lambda_{\beta\gamma]} \,, \\
\delta\phi &= \mathcal{L}_\xi \phi = \xi^\gamma \partial_\gamma \phi \,.
\end{aligned}
\tag{4}
$$

- The diffeo and gauge transformation generators combine in a generalised vector in **10**, U^{ab}:

$$
\begin{aligned}
U^{\alpha 5} &= \xi^\alpha \,, \\
U^{\alpha\beta} &= \frac{1}{2}\eta^{\alpha\beta\gamma\delta}\lambda_{\gamma\delta} \,.
\end{aligned}
\tag{5}
$$

- The combined action is given by generalised Lie derivative (Berman, Godazgar, Perry '11):

$$
L_U V^a = \frac{1}{2} U^{bc}\partial_{bc} V^a + \frac{1}{4} V^a \partial_{bc} U^{bc} - V^b \partial_{bc} U^{ac} \,, \tag{6}
$$

when we take $\partial^{\alpha\beta} = \frac{\partial}{\partial y_{\alpha\beta}} = 0$.

Section condition

- Consistency of the theory ⇒ algebra of generalised diffeomorphisms must close.
- But we find:

$$\left[L_U, L_V\right] W^A = L_{[U,V]_C} W^A + \text{junk} \,. \tag{7}$$

- Junk vanishes when we impose the "section condition":

$$\partial_{[ab} f \, \partial_{cd]} g = 0 \,. \tag{8}$$

  for all fields f, g in the theory. (Berman, Godazgar, Perry '11)
- In general EFT is not consistent but can be made consistent by imposing section condition: it is a constrained theory.
- Section condition is $\mathrm{SL}(5)$ covariant requirement that winding derivatives vanish: $\partial^{\alpha\beta} = 0$, i.e. dependence only on 4 coordinates.

The section and the action

Requiring an action that is invariant under generalised diffeomorphisms gives the unique answer (Berman, Perry '10)

$$S = \int_\Sigma |m|^{-1} \Big(\frac{1}{8} m^{ab} m^{a'b'} \partial_{aa'} m^{cd} \partial_{bb'} m_{cd} - \frac{1}{2} m^{ab} m^{a'b'} \partial_{aa'} m^{cd} \partial_{cb'} m_{bd} \\ - \frac{1}{2} \partial_{aa'} m^{ab} \partial_{bb'} m^{a'b'} - \frac{3}{8} m^{ab} m^{a'b'} \partial_{aa'} \ln|m| \, \partial_{bb'} \ln|m| \\ + 2 m^{a'b'} \partial_{aa'} m^{ab} \partial_{bb'} \ln|m| - m^{a'b'} \partial_{aa'} \partial_{bb'} m^{ab} + m^{ab} m^{a'b'} \partial_{aa'} \partial_{bb'} \ln|m| \Big) \tag{9}$$

- Restricting to the section $\partial^{\alpha\beta} = 0$, $\partial_\alpha \neq 0$ the action reduces to scalar potential of 11-d supergravity

$$S = \int d^4x \; e^{7\phi/2} \sqrt{|g|} \left(R - \frac{1}{48} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} + \frac{21}{2} \partial^\alpha \phi \partial_\alpha \phi \right) . \tag{10}$$

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IIB theory

The section condition

$$\partial_{[ab} f \partial_{cd]} g = 0 \,, \tag{11}$$

has a different inequivalent solution: Fields only depend on three coordinates (Blair, EM, Park '13)

$$x_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho} X^{\nu\rho} \,, \qquad \mu, \nu = 1, 2, 3. \tag{12}$$

Decomposing the $\mathrm{SL}(5) \longrightarrow \mathrm{SL}(3) \times \mathrm{SL}(2)$, m_{ab} in **15** of $\mathrm{SL}(5)$ decomposes to $(\mathbf{6}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1})$.

These correspond to

- 3×3 symmetric $\tilde{g}^{\mu\nu}$ spacetime metric, $|\tilde{g}| = |\det \tilde{g}^{\mu\nu}|$,
- 2 vectors $\tilde{v}^i{}_\mu$ (dual to KR and RR 2-forms), $i = 4, 5$ is $\mathrm{SL}(2)$ index,
- 3 scalars: $\tilde{\phi}$, symmetric 2×2 unit-det $\tilde{\mathcal{M}}_{ij}$ (dilaton and RR 0-form)

$$\tilde{\mathcal{M}}_{ij} = \frac{1}{\mathrm{Im}\tau} \begin{pmatrix} |\tau|^2 & \mathrm{Re}\tau \\ \mathrm{Re}\tau & 1 \end{pmatrix} \,, \quad \tau = C^{(0)} + i e^{-\tilde{\varphi}} \,. \tag{13}$$

IIB symmetries

- Natural parameterisation of m_{ab} is

$$m_{ab} = \begin{pmatrix} \sqrt{|\tilde{g}|}\left(\tilde{g}_{\mu\nu} + e^{\tilde{\phi}}\tilde{v}^{k}{}_{\mu}\tilde{v}_{k\nu}\right) & e^{\tilde{\phi}}\tilde{v}_{j\mu} \\ e^{\tilde{\phi}}\tilde{v}_{i\nu} & \frac{1}{\sqrt{|\tilde{g}|}}e^{\tilde{\phi}}\tilde{\mathcal{M}}_{ij} \end{pmatrix} . \qquad (14)$$

- Generalised vector in **10**: $U^{ab} = \tilde{\xi}_{\mu} \oplus \lambda^{i\mu} \oplus \xi^{ij}$.
- Generalised Lie derivative gives gauge transformations:

$$\begin{aligned} \delta\tilde{\phi} &= \mathcal{L}_{\tilde{\xi}}\tilde{\phi}\,, & \delta\tilde{\mathcal{M}}_{ij} &= \mathcal{L}_{\tilde{\xi}}\tilde{\mathcal{M}}_{ij}\,, \\ \delta\tilde{C}^{i\mu\nu} &= \mathcal{L}_{\tilde{\xi}}\tilde{C}^{i\mu\nu} + 2\tilde{\partial}^{[\mu}\lambda^{|i|\nu]}\,, & \delta\tilde{g}^{\mu\nu} &= \mathcal{L}_{\tilde{\xi}}\tilde{g}^{\mu\nu}\,. \end{aligned} \qquad (15)$$

We defined $\mathcal{L}_{\tilde{\xi}}V^{\mu} \equiv \tilde{\xi}_{\nu}\tilde{\partial}^{\nu}V^{\mu} + V^{\nu}\tilde{\partial}^{\mu}\tilde{\xi}_{\nu}$,

- ξ^{ij} drops out of Lie derivative (no gauge transformation of $C^{(0)}$).

IIB action

Using the IIB parameterisation and section condition, the EFT action reduces to

$$S = \int d^3\tilde{x} \sqrt{|\tilde{g}|} e^{-2\tilde{\phi}} \left(\tilde{R} + \frac{1}{4} \tilde{\partial}^\mu \tilde{\mathcal{M}}^{ij} \tilde{\partial}_\mu \tilde{\mathcal{M}}_{ij} - \frac{1}{12} e^{\tilde{\phi}} \tilde{H}^{i\mu\nu\rho} \tilde{H}_{i\mu\nu\rho} \right. \\ \left. + \frac{9}{2} \tilde{\partial}^\mu \tilde{\phi} \tilde{\partial}_\mu \tilde{\phi} \right) .$$

- μ, ν indices raised/lowered by $\tilde{g}^{\mu\nu}$,
- i, j indices are raised/lowered by $\tilde{\mathcal{M}}_{ij} \Rightarrow$ manifestly $\mathrm{SL}(2)$ invariant,
- $\mathrm{SL}(2)$ doublet of field strengths $\tilde{H}^{i\mu\nu\rho} = 3\tilde{\partial}^{[\mu} \tilde{C}^{|i|\nu\rho]}$,
- Ricci scalar $\tilde{R}$ for $\tilde{g}^{\mu\nu}$ with "reversed indices".
- This is 7-d scalar potential of IIB theory with indices reversed.

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Geometry of EFT

The action for EFT is a great success:

- it is $\mathrm{SL}(5)$ invariant,
- it reduces to 7-d scalar potential of 11-d when using $\partial^{\alpha\beta} = 0$,
- it reduces to 7-d scalar potential of IIB supergravity action when using $\partial_{\mu i} = \partial_{ij} = 0$,
- it is a generalised scalar up to section condition.
- However, invariance under generalised diffeos is not apparent from its form:

$$S = \int (\partial m)^2 \ . \tag{16}$$

Wish list

Ideally, we want a connection for EFT to satisfy

- covariant derivative maps generalised tensors to generalised tensors,
- compatible with m_{ab},
- compatible with the $\mathrm{SL}(5)$ invariant ϵ_{abcde},
- completely determined in terms of physical fields,
- torsion-free,
- gives a curvature tensor that may be contracted with generalised metric to give the scalar appearing in action.

Wish list

Ideally, we want a connection for EFT to satisfy

- covariant derivative maps generalised tensors to generalised tensors, ✓
- compatible with m_{ab}, ✓
- compatible with the $\mathrm{SL}(5)$ invariant ϵ_{abcde}, ✓
- completely determined in terms of physical fields, ✗
- torsion-free, ✓
- gives a curvature tensor that may be contracted with generalised metric to give the scalar appearing in action. ??✓??
  Aldazabal et al '13, Cederwall et al '13, Hohm et al '13, Park et al '14

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- covariant derivative maps generalised tensors to generalised tensors, ✓
- compatible with m_{ab}, ✓
- compatible with the $\mathrm{SL}(5)$ invariant ϵ_{abcde}, ✓
- completely determined in terms of physical fields, ✓
- torsion-free, ✗
- gives a curvature tensor that may be contracted with generalised metric to give the scalar appearing in action. ✗
  Blair, EM '14

Flux formulation

- Because gen. diffeos are not just diffeos of the enlarged space, we cannot just use a Levi-Civita type object.
- We use a flat ("Weitzenbck") connection.

$$\nabla_{ab} V^c = \partial_{ab} V^c + \Gamma^c_{ab,d} V^d \,, \qquad \Gamma^c_{ab,d} = E_{\bar{a}}{}^c \partial_{ab} E^{\bar{a}}{}_d \,. \tag{17}$$

- $E^{\bar{a}}{}_a$ are generalised vielbeine: $E^{\bar{a}}{}_a E_{\bar{a}b} = m_{ab}$. ($\bar{a}, \bar{b}, \ldots$ are flat $\mathrm{SL}(5)$ indices.)
- The spin connection vanishes:

$$\nabla_{ab} V^{\bar{a}} = \partial_{ab} V^{\bar{a}} \,. \tag{18}$$

- In EFT we cannot define a Riemann-curvature-like object (Cederwall et al '13).

Torsion

- Use torsion instead:

$$\left(L_U^{\nabla} - L_U^{\partial}\right) V^a = \frac{1}{2}\tau_{bc,d}{}^a U^{bc} V^d \,. \tag{19}$$

- The torsion is given by

$$\tau_{bc,d}{}^a = 3\Gamma^a_{[bc,d]} - \Gamma^e_{e[b,c]}\delta^a_d - 2\Gamma^e_{ed,[b}\delta^a_{c]} \,. \tag{20}$$

- Connection and torsion are not invariant under local $\mathrm{SO}(5)$.

$$\begin{aligned} \Gamma^c_{ab,d} &= E_{\bar{a}}{}^c \partial_{ab} E^{\bar{a}}{}_d \,. \\ \delta_\lambda \Gamma^c_{ab,d} &= E_{\bar{a}}{}^c E_{\bar{b}d} \partial_{ab} \lambda^{\bar{a}\bar{b}} \,. \end{aligned} \tag{21}$$

- ⇒ Find the action by requiring invariance under $\mathrm{SO}(5)$.

Torsion Irreducibles

- Find the action in terms of torsion by requiring $\mathrm{SO}(5)$ invariance.
- First, decompose torsion into its irreps

$$\tau_{bc,d}{}^{a} = \frac{1}{2}\epsilon_{bcdef} Z^{ef,a} + \frac{1}{2}\delta^{a}_{[b} S_{c]d} + \frac{1}{3}\delta^{a}_{d}\tau_{bc,e}{}^{e} + \frac{2}{3}\delta^{a}_{[b}\tau_{c]d,e}{}^{e} \qquad (22)$$

- $\overline{\mathbf{40}}$: $Z^{ab,c} = Z^{[ab],c}$ with $Z^{[ab,c]} = 0$,
- $\mathbf{15}$: $S_{ab} = S_{(ab)}$,
- $\mathbf{10}$: $\tau_{ab,e}{}^{e}$.
- Write all terms that involve these irreps squared $\left(m^{ab} S_{ab}\right)^2$, etc.
- Compute $\mathrm{SO}(5)$ variation of these terms and choose combination so action is invariant.

The action

- We find the unique $\mathrm{SO}(5)$ invariant combination

$$\begin{aligned} S = \int_\Sigma dX & \frac{1}{16} m^{ab} m^{cd} S_{ac} S_{bd} - \frac{1}{32} \left(m^{ab} S_{ab} \right)^2 + \frac{5}{3} m^{ab} m^{cd} \tau_{ac,e}{}^{e} \tau_{bd,f}{}^{f} \\ & + \frac{1}{4} m_{ab} m_{cd} m_{ef} Z^{ce,a} Z^{df,b} - \frac{3}{8} m_{ab} m_{cd} m_{ef} Z^{ce,[a} Z^{df],b} \\ & - 2 m^{ab} m^{cd} \nabla_{ac} \tau_{bd,e}{}^{e} , \end{aligned} \qquad (23)$$

  where Σ is a section satisfying section condition.
- This is manifestly generalised diffeo invariant.
- This is $\mathrm{SO}(5)$ invariant up to section condition.
- Reduces to EFT action when section condition is imposed.
- Extra term is crucial to obtain action for gauged SUGRA.

The connection to gauged SUGRA

What about gauged SUGRA? (Berman, Musaev, Thompson '12, Hohm, Samtleben '14)

- Standard Kaluza-Klein (flat reduction): no dependence on internal EFT coordinates X^{ab}.
- Twisted (Scherk-Schwarz) compactifications: dependence on X^{ab} is carried by $\mathrm{SL}(5)$ group element $E_a{}^{\bar{a}}$.
- 7-d fields are twisted, e.g. $g_7\left(X,y\right) = |E|^{-1}(X)\hat{g}_7\left(y\right)$.
- The generalised metric $m_{ab} = E_a{}^{\bar{a}} E_{b\bar{a}}$ describes the compactification space: metric, 3-form and warp factor.
- The (flattened) torsion reduces to the structure constants of the gauged SUGRA

$$[E_{\bar{a}\bar{b}}, E_{\bar{c}}] = f_{\bar{a}\bar{b},\bar{c}}{}^{\bar{d}} E_{\bar{d}} = \tau_{\bar{a}\bar{b},\bar{c}}{}^{\bar{d}} E_{\bar{d}} \,. \tag{24}$$

Gauged SUGRA

- For the flat connection we find
$$L_U V^a = E_{\bar{a}}{}^a \left(L_U V^{\bar{a}} - \frac{1}{2} \tau_{\bar{b}\bar{c},\bar{d}}{}^{\bar{a}} U^{\bar{b}\bar{c}} V^{\bar{d}} \right) , \tag{25}$$
where $L_U V^{\bar{a}}$ is the generalised Lie derivative acting on flat indices.
- Thus, we find the local symmetries of the gauged SUGRA given by
$$\hat{L}_U V^{\bar{a}} = L_U V^{\bar{a}} - \frac{1}{2} \tau_{\bar{b}\bar{c},\bar{d}}{}^{\bar{a}} U^{\bar{b}\bar{c}} V^{\bar{d}} . \tag{26}$$
- This defines the embedding tensor $\Theta = \tau$!
- The action reduces to the action for 7-d maximal gauged SUGRA.
- Embedding tensor is subject to "linear constraint": it must only contain irreps $\mathbf{15} \oplus \overline{\mathbf{40}}$ of $\mathrm{SL}(5)$.
- This is (almost) satisfied automatically by the torsion.
- We need to impose **10** vanishes (trombone gauging)?

Section condition revisited

- Closure of algebra of Lie derivatives led to section condition.
- Let us revisit this in the Scherk-Schwarz setup. We find:

$$[\tau_{\bar{a}\bar{b}}, \tau_{\bar{c}\bar{d}}] = \tau_{\bar{c}\bar{d},[\bar{a}}{}^{\bar{e}}\tau_{\bar{b}]\bar{e}} - \tau_{\bar{a}\bar{b},[\bar{c}}{}^{\bar{e}}\tau_{\bar{d}]\bar{e}}\,, \tag{27}$$

  here we denote by $\tau_{\bar{a}\bar{b}}$ the matrix with element $(\tau_{\bar{a}\bar{b}})_{\bar{c}}{}^{\bar{d}} = \tau_{\bar{a}\bar{b},\bar{c}}{}^{\bar{d}}$.
- This is exactly the quadratic constraint of gauged SUGRA.
- Section condition solves this, i.e. section condition ⇒ quadratic constraint.
- This has solutions which violate the section condition. What do these mean? Beyond SUGRA? EFT still consistent. (Waldram et al '15)
- It can be shown that the constraint (27) and having constant torsion is enough for a consistent compactification, i.e. solutions of the gauged SUGRA are solutions of the full 11-d or 10-d theory. (Berman, Musaev, Thompson '12, Hohm, Samtleben '14)

A quick break: summary so far

- We have shown that $\mathrm{SL}(5)$ EFT contain 11-d as well as IIB theory.
- We have described a geometry for $\mathrm{SL}(5)$ EFT. This was based on a flat connection.
- We found torsion gives higher-dimensional origin to embedding tensor of gauged SUGRAs.
- Linear constraint automatically satisfied, quadratic constraint satisfied by section condition. (Possible relaxation of section condition?)
- EFT twists that satisfy quadratic constraint (section condition) and have constant flattened torsion automatically lead to consistent compactifications.
- Next: Sphere and hyperboloid compactifications as EFT twists.

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$\mathrm{SO}(p, q)$ gaugings

- Sphere compactifications can be elegantly described by an EFT twist, e.g. S^4 and S^3 compactifications of 11-d and 10-d IIA SUGRA Hohm, Samtleben '14 (Godazgar^2, Nicolai '13, Baron, Dall'Agata '14 for 4-d).
- The consistency of these compactifications follows straightforwardly using EFT.
- The internal space can be read off from the generalised metric $m_{ab} = E^{\bar{c}}{}_a E_{\bar{c}b}$, where E is the twist matrix used.
- The complicated non-linear Ansätze for the internal metric and form-fields follows from a simple EFT twist.
- The compactifications can be generalised to *hyperboloids* to obtain non-compact $\mathrm{SO}(p, q)$ gaugings (Hohm, Samtleben '14, Baron, Dall'Agata '14).

$\mathrm{SO}(p,q)$ gaugings of IIA

- Hohm, Samtleben '14 discuss $H^{p,q}$ compactifications of IIA (11-d) SUGRA to 7-d maximal gauged SUGRA using EFT.
- $H^{p,q}$ is a constant curvature space with p positive and q negative directions, i.e. $p=4$, $q=0$ is a S^4, $p=0$, $q=4$ is H^4, etc.
- These compactifications turn on the **15** of the embedding tensor $S_{(\bar{a}\bar{b})}=\eta_{\bar{a}\bar{b}}$, where $\eta_{\bar{a}\bar{b}}$ is the $\mathrm{SO}(p,q)$ invariant.
- For the compactification of IIA, decompose the $\mathrm{SL}(5)$ to $\mathrm{SL}(4)\sim\mathrm{SO}(3,3)$ (the 3-d T-duality group).
- $S_{\bar{a}\bar{b}}$ then decomposes to $\mathbf{10}\oplus\mathbf{4}\oplus\mathbf{1}$ of $\mathrm{SL}(4)$.
- The IIA $H^{p,3-p}$ compactifications give $\mathrm{SO}(p,4-p)$ gaugings in the **10**.
- This is described by a $\mathrm{SO}(p,4-p)$ tensor $S_{\bar{\alpha}\bar{\beta}}=\eta_{\bar{\alpha}\bar{\beta}}$ where $\bar{\alpha},\bar{\beta}=1,\ldots 4$.

IIA and IIB gaugings

- Recall that the embedding tensor has another irrep, the $\overline{\mathbf{40}}$.
- Decomposing $\overline{\mathbf{40}}$ under $\mathrm{SL}(4)$ we have $\overline{\mathbf{20}} \oplus \overline{\mathbf{10}} \oplus \mathbf{6} \oplus \overline{\mathbf{4}}$.
- The $\overline{\mathbf{10}}$ gauging can again be described by a symmetric tensor $\eta^{\bar{\alpha}\bar{\beta}}$ which defines a $\mathrm{SO}(p, q)$ gauging.
- It is expected that this comes from a $H^{p,3-p}$ reduction of the IIB theory.
- Is there something like a "duality" that turns a IIA gauging of **10** into a IIB gauging of $\overline{\mathbf{10}}$?
- This would have to exchange the internal coordinates corresponding to IIA and IIB i.e. act like a $\mathrm{O}(3,3)$ transformation of negative determinant.
- This cannot be a $\mathrm{SL}(5)$ transformation since this reduces to $\mathrm{SL}(4) \sim \mathrm{SO}(3,3)$.

IIA and IIB gaugings

- Recall that the **15** ($S_{\bar{a}\bar{b}}$) and $\overline{\mathbf{40}}$ ($Z^{\bar{a}\bar{b},\bar{c}}$) are defined in terms of the Weitzenböck connection of the twist E.
- Study compactifications of 10-d SUGRA (both IIA and IIB) $\Rightarrow \partial_{\alpha 4} = 0$, but no section condition (yet).
- Using the Ansatz

$$E_{\bar{a}}{}^{b} = \rho^{-1/2} \begin{pmatrix} \omega^{1/2} V_{\bar{\alpha}}{}^{\beta} & 0 \\ 0 & \omega^{-2} \end{pmatrix} , \qquad (28)$$

we find that we must have $\rho = \omega$ and the only non-zero gaugings to be the $\mathbf{10} \subset \mathbf{15}$, $\overline{\mathbf{10}}\,\&\,\mathbf{6} \subset \overline{\mathbf{40}}$ and $\mathbf{6} \subset \mathbf{10}$, given by:

$$\begin{aligned} S_{\bar{\alpha}\bar{\beta}} &= 4 V_{(\bar{\alpha}}{}^{\alpha} \partial_{|\alpha\beta|} V_{\bar{\beta})}{}^{\beta} , \qquad Z^{\bar{4}(\bar{\alpha},\bar{\beta})} = V_{\alpha}{}^{(\bar{\alpha}} \partial^{|\alpha\beta|} V_{\beta}{}^{\bar{\beta})} , \\ 6 Z^{\bar{4}[\bar{\alpha},\bar{\beta}]} &= \left(\partial^{\alpha\beta} V_{\alpha\beta}{}^{\bar{\alpha}\bar{\beta}} - 5 V_{\alpha\beta}{}^{\bar{\alpha}\bar{\beta}} \partial^{\alpha\beta} \ln \omega \right) , \qquad (29) \\ 2 \tau_{\bar{\alpha}\bar{\beta},\bar{c}}{}^{\bar{c}} &= \left(-\partial_{\alpha\beta} V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta} + 5 V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta} \partial_{\alpha\beta} \ln \omega \right) , \end{aligned}$$

where we define $\partial^{\alpha\beta} \equiv \frac{1}{2} \eta^{\alpha\beta\gamma\delta} \partial_{\gamma\delta}$.

IIA and IIB gaugings

$$S_{\bar{\alpha}\bar{\beta}} = 4V_{(\bar{\alpha}}{}^{\alpha}\partial_{|\alpha\beta|}V_{\bar{\beta})}{}^{\beta}\,, \qquad Z^{\bar{4}(\bar{\alpha},\bar{\beta})} = V_{\alpha}{}^{(\bar{\alpha}}\partial^{|\alpha\beta|}V_{\beta}{}^{\bar{\beta})}\,, \tag{30}$$

- It is clear that we can turn a gauging of the $\mathbf{10} \subset \mathbf{15}$ into a $\overline{\mathbf{10}} \subset \overline{\mathbf{40}}$ by the outer automorphism of $\mathrm{SL}(4)$

$$V \longleftrightarrow V^{-T}\,, \qquad \partial_{\alpha\beta} \longleftrightarrow \partial^{\alpha\beta}\,. \tag{31}$$

- Recall that $\mathrm{SL}(4) \sim \mathrm{SO}(3,3)$, the 3-d T-duality group. The outer automorphism corresponds to the $\mathrm{O}(3,3)$ transformation not in $\mathrm{SO}(3,3)$, i.e. swaps IIA and IIB.
- Indeed if we have a twist that only depends on the coordinates $X^{\mu 5}$, $\mu = 1,2,3$ (i.e. coming from IIA), this gets turned into a twist that only depends on $X^{\mu\nu}$, i.e. a IIB compactification.
- This automorphism also swaps the $\mathbf{6} \subset \mathbf{10}$ and $\mathbf{6} \subset \overline{\mathbf{40}}$.
- It can be shown that with this Ansatz, the $\mathbf{6}$ are identical $\Rightarrow$ if we don't want trombone gauging ($\mathbf{10}$ of $\mathrm{SL}(5)$), we must not have $\mathbf{6} \subset \overline{\mathbf{40}}$.

IIB $\mathrm{SO}(p,q)$ gaugings

- Using this map and the IIA twists, we can find IIB compactifications to 7-d SUGRA corresponding to $\mathrm{SO}(p,q)$ gaugings.
- For the internal space we find a sphere or hyperboloid. In string frame,

$$ds_4^2 = (1+u-v)^{-1}\, dx_\mu dx_\nu \left(\delta^{\mu\nu} + \frac{1}{1-v} \eta^{\mu\rho}\eta^{\nu\sigma} x_\rho x_\sigma \right) , \qquad (32)$$

  where $x_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho}X^{\nu\rho}$, $u = x^\mu x_\mu$, $v = x_\mu x_\nu \eta^{\mu\nu}$ and $\eta^{\mu\nu}$ is the pull-back of the $\overline{\mathbf{10}}$ gauging $Z^{\bar{4}(\bar{\alpha},\bar{\beta})}$ to the $\mu,\nu = 1,\ldots,3$ directions.
- We also find a KR 2-form in the internal directions.
- The metric and KR 2-form coincides with that for the IIA compactification.

Gauging the $\overline{\mathbf{4}} \subset \overline{\mathbf{40}}$

- It is also easy to find a compactification that additionally gauges the $\overline{\mathbf{4}} \subset \overline{\mathbf{40}}$.
- We need to generalise the Ansatz for the twist to

$$E_{\bar{a}}{}^{b} = \omega^{-1/2} \begin{pmatrix} \omega^{1/2} V_{\bar{\alpha}}{}^{\beta} & -A_{\bar{\alpha}}\omega^{-2} \\ 0 & \omega^{-2} \end{pmatrix} . \qquad (33)$$

- We still keep $\partial_{\alpha 4} = 0$ so that the reduction is from 10-d.
- The **10**'s and **6**'s are unchanged but we also find for the $\overline{\mathbf{4}} \subset \overline{\mathbf{40}}$:

$$Z^{\bar{4}\bar{\alpha},\bar{4}} = -V_{\alpha}{}^{\bar{\alpha}} \partial^{\alpha\beta} A_{\beta} + \frac{5}{2} V_{\alpha}{}^{\bar{\alpha}} A_{\beta} \partial^{\alpha\beta} \ln \omega \,, \qquad (34)$$

where $A_{\alpha} = V_{\alpha}{}^{\bar{\beta}} A_{\bar{\beta}}$.

- For the case when $Z^{\bar{4}(\bar{\alpha},\bar{\beta})} = \eta^{\bar{\alpha}\bar{\beta}}$ is invertible, we can solve this with

$$A_{\bar{\alpha}} = \eta_{\bar{\alpha}\bar{\beta}} d^{\bar{\beta}} \,, \qquad (35)$$

for some constant $d^{\bar{\alpha}}$ to find $Z^{\bar{4}\bar{\alpha},\bar{4}} = d^{\bar{\alpha}}$.

Gauging the $\overline{\mathbf{4}} \subset \overline{\mathbf{40}}$

- Let us calculate the internal space from gen metric m_{ab}.
- We find metric and KR 2-form are unchanged.
- We find *additional* RR 2-form and 0-form:

$$C_0 = a \,, \qquad \tilde{V}_\mu{}^4 = (1+u-v)^{-3/8} \left\{ \eta_{\mu\nu} \left[(1+u-v)\, d^\nu + x^\nu a \right] \right\} \,, \qquad (36)$$

where $a = -(d \cdot x) + (1-v)^{3/4}\, \eta_{55} d^5$.

- Different solutions to the gauging equation represent gauge transformations of the R-R sector:

$$-V_\alpha{}^{\bar\alpha} \partial^{\alpha\beta} \delta A_\beta + \frac{5}{2} V_\alpha{}^{\bar\alpha} \delta A_\beta \partial^{\alpha\beta} \ln \omega = 0 \,. \qquad (37)$$

- This is a new consistent compactification to d=7.
- It is a compactification of IIB on $H^{p,q}$ with R-R fields turned on.

Outline

1. References
2. 11-d SUGRA, U-dualities and gauged SUGRAs
3. Exceptional field theory
4. IIB theory
5. A geometry: flux formulation
6. SO(p, q) gaugings
7. Summary
8. Further work

Summary 2

- Introduced EFT and showed it reduces to 11-d and IIB SUGRA.
- EFT is only consistent up to section condition.
- Constructed a geometry based on “flux formulation” for $\mathrm{SL}(5)$ EFT.
- Close connection to gauged SUGRA: torsion is embedding tensor.
- Discussed sphere and hyperboloid compactifications as twisted EFT reductions.
- Showed how to generate IIB compactifications from IIA compactifications.
- Discussed some new compactifications.

Outline

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Further work

- Extend the map from IIA and IIB to more general twist Ansatz.
- Find consistent compactifications of IIA or IIB that gauges the $\mathbf{6} \subset \overline{\mathbf{40}}$?
- Find consistent compactifications that gauge the trombone?
- Find more consistent compactifications of IIA and IIB to obtain more of the spectrum of allowed gaugings?
- Extension to lower dimensions (higher exceptional groups).
- Less SUSY.
- Find a higher dimensional origin for orphaned gauged SUGRAs, e.g. using E_7 for deformed $\mathrm{SO}(8)$ theories (Dall'Agata et al '12).
- Violate section condition (Waldram et al '15).

Thank you!

Thank you for listening!