

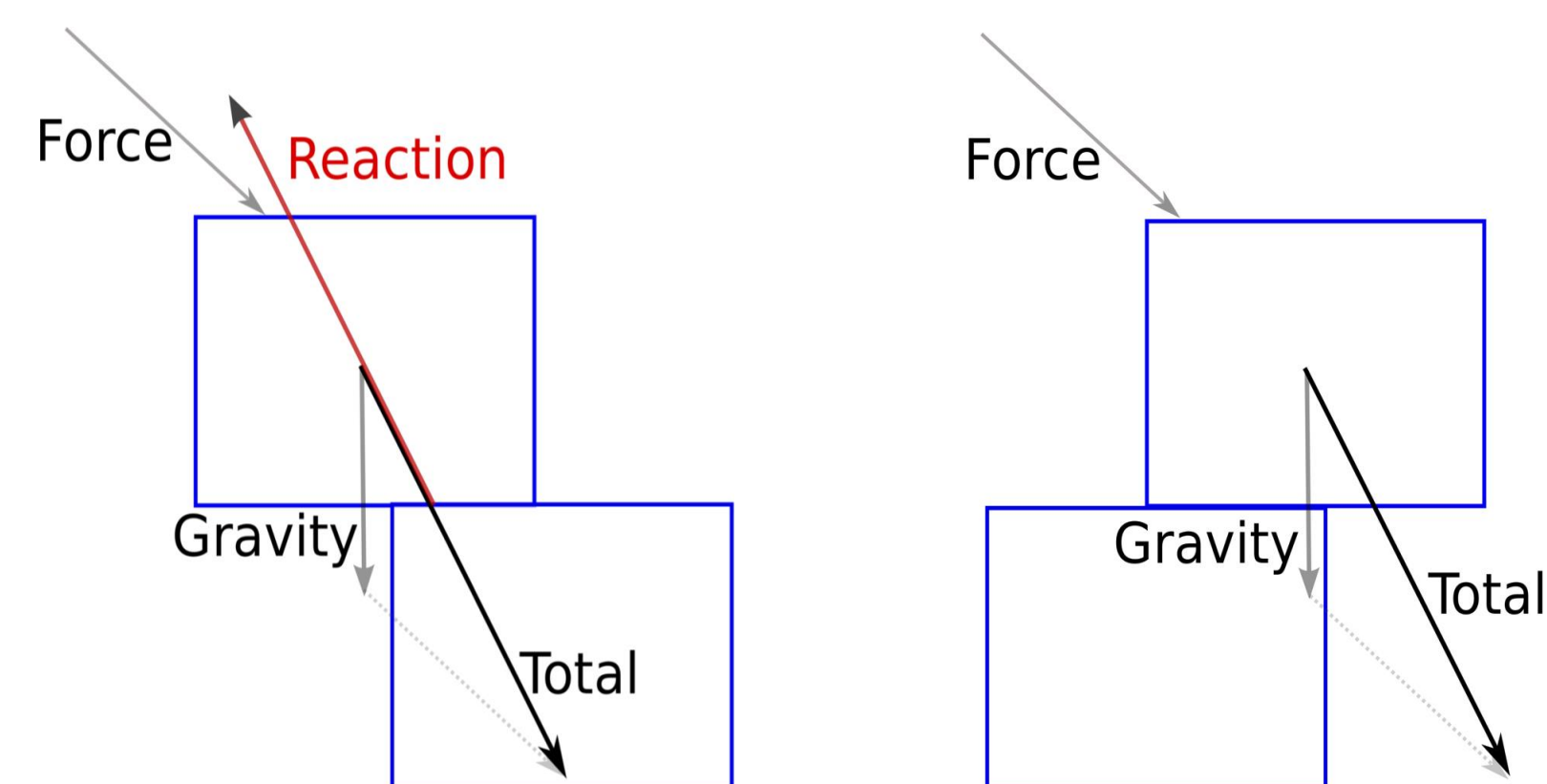
Creating Cathedrals

The maths behind arches and cathedrals

Building an arch

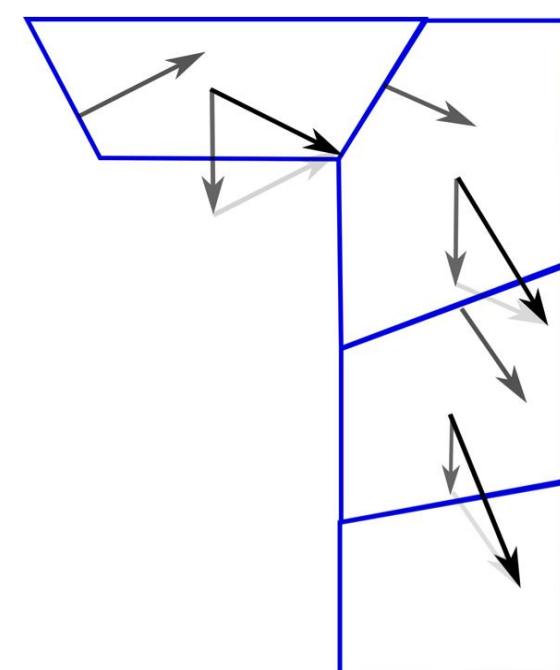
Cathedrals are the largest middle-age structures. They don't require steel or other modern reinforcing. In this poster we will try to understand the physics behind these constructions.

A **force diagram** shows the forces acting on an object. Forces have a magnitude and direction, which are shown using arrows or vectors. For an object to be in equilibrium, there must also exist a reaction force which is equal and opposite to the total of the acting forces. The reaction force acts on the centre of mass.



For the picture above left, the total force lies within the object and so the (red) reaction force can be exerted by the bottom block. This means the object is in equilibrium. For the picture above right this is not the case as the total force lies outside of the object. You can see this example by putting a glass at the edge of a table. If you put the glass further than the centre of mass then the glass will fall off since there is nothing to provide the reaction force.

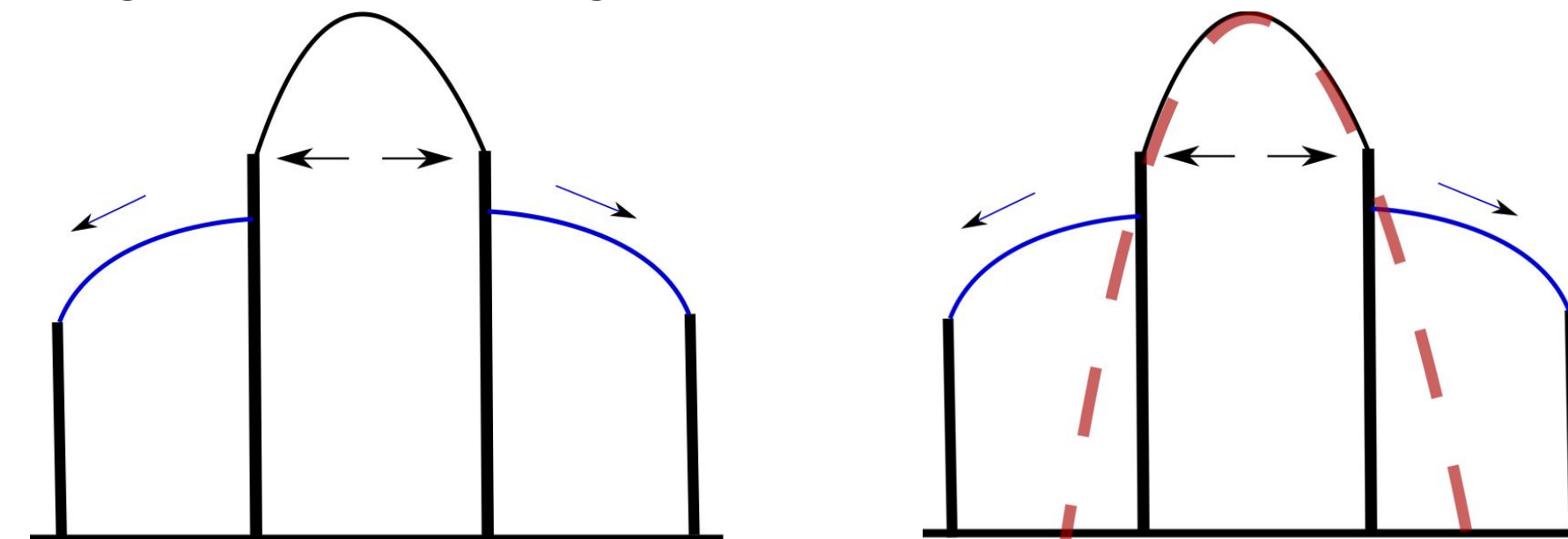
For an arch in a cathedral to stand, we need each block to be in equilibrium. This means the sum of the forces acting on each block needs to lie inside the arch.



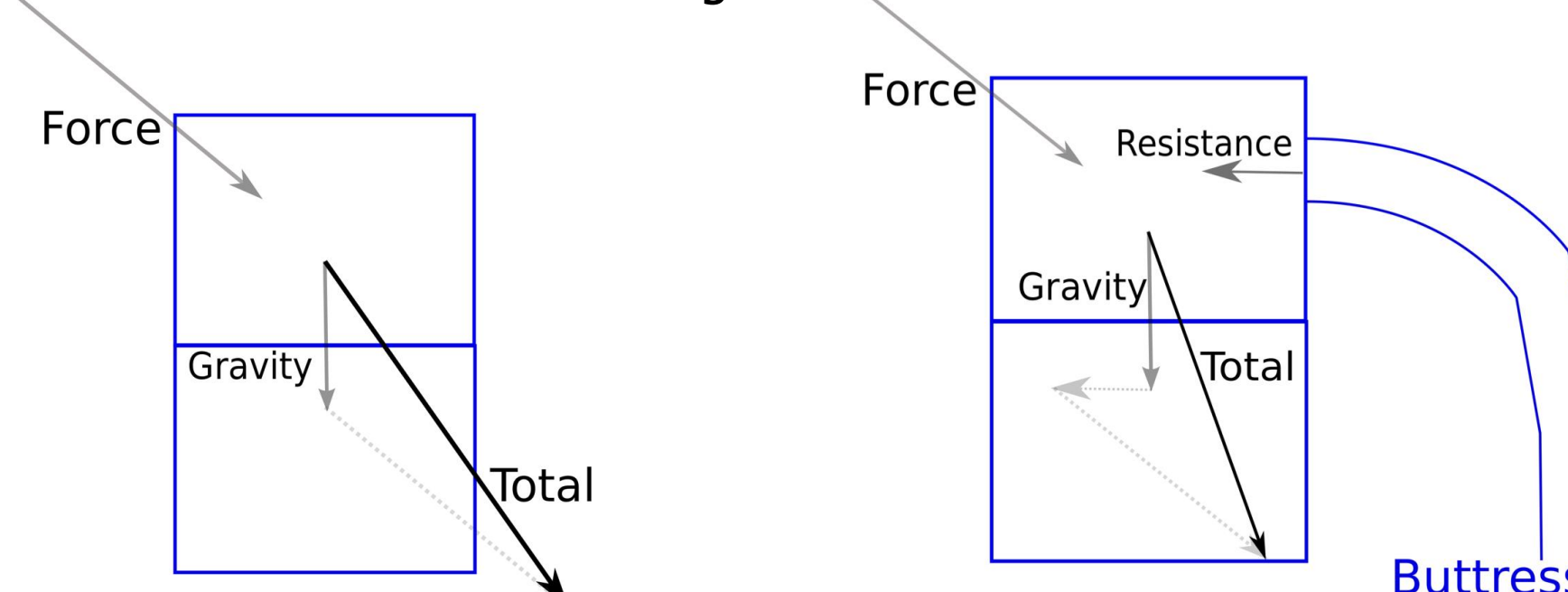
We only need a single point of contact, which is exactly where the reaction forces goes through. The reaction forces defines the **line of thrust** of the arch.

Flying Buttresses

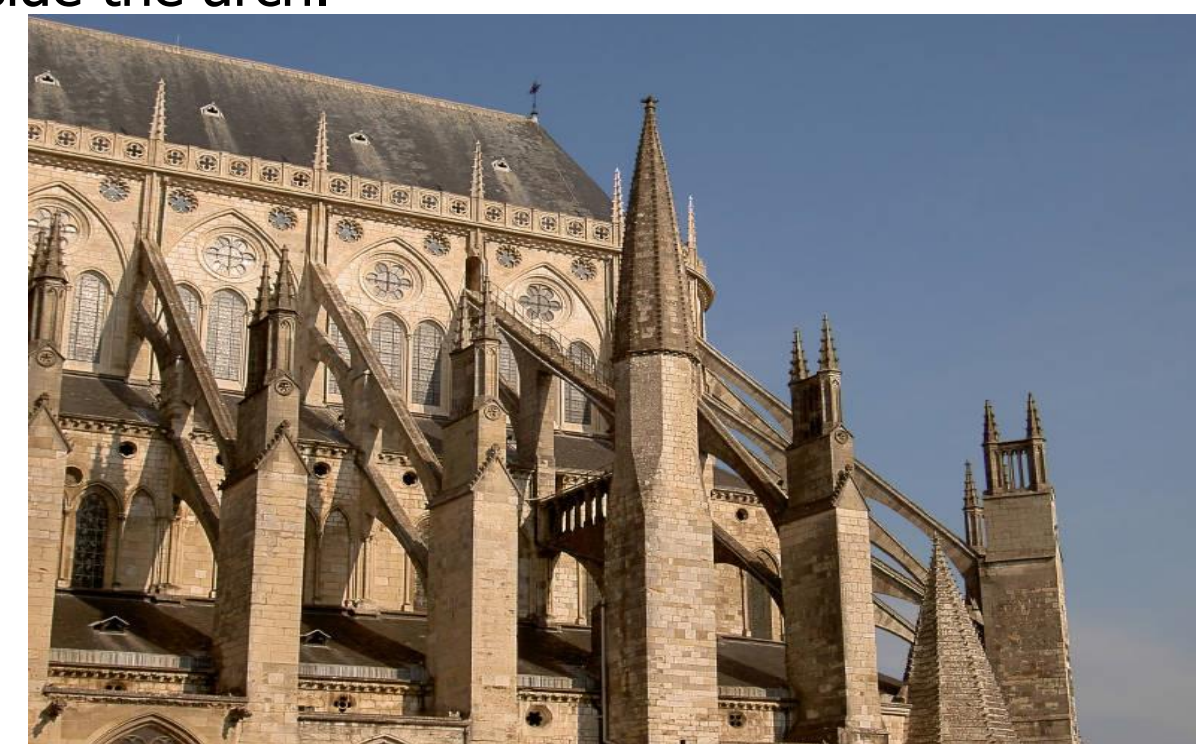
To build higher arches, builders need to widen the walls of the arch so that the line of force remains within the arch. However walls can only be so thick, so this limits the height of cathedrals. To overcome this problem architects employ the **flying buttress**. The flying buttress allows the line of force (shown in red below right) of the vault (roof) to be brought down to the ground while allowing for thinner walls.



Another benefit of flying buttresses is that cathedrals can now have side aisles outside of the main walls. The flying buttress brings the force from the vault down over the aisles to the ground.



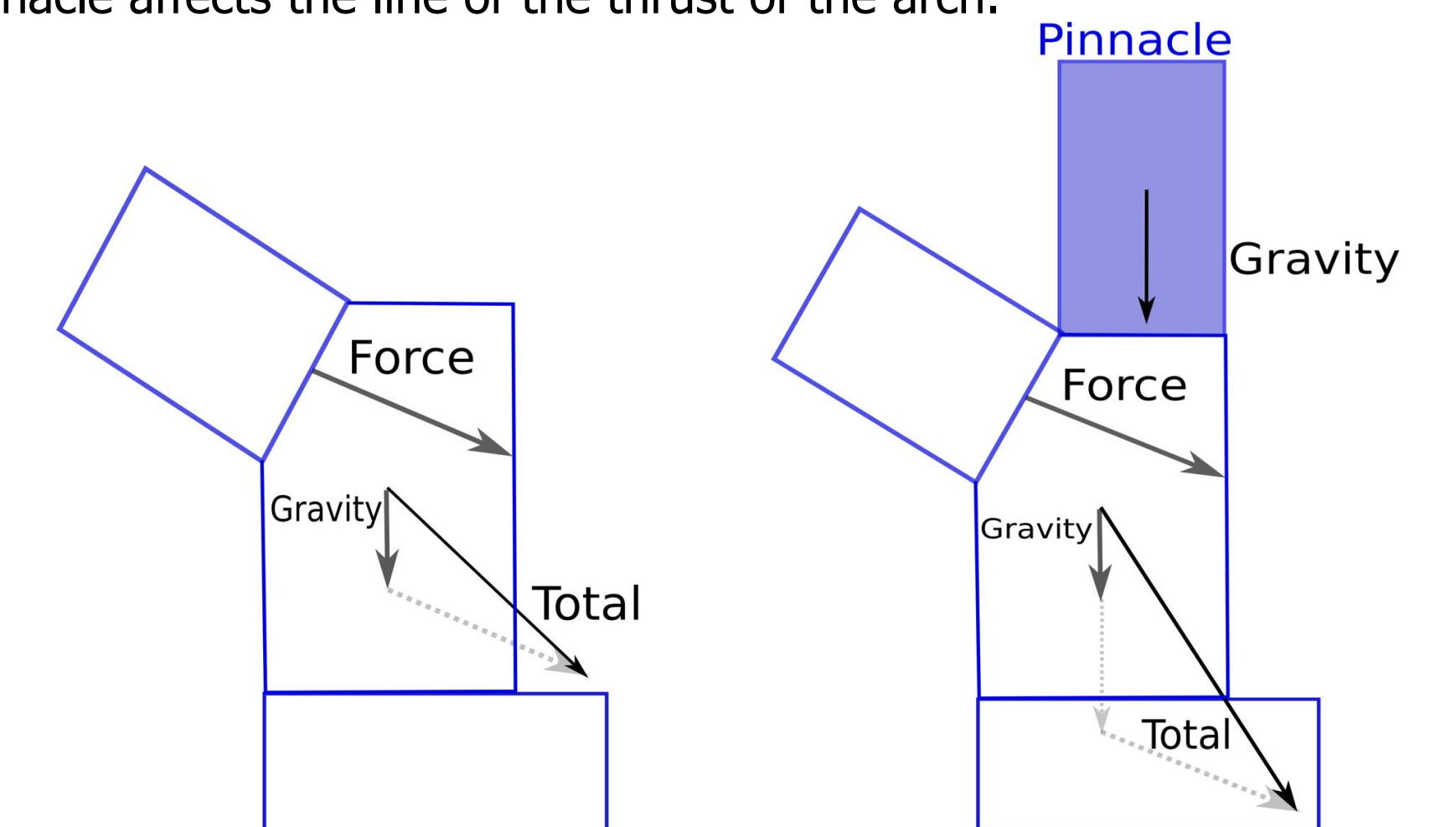
In the above left picture, the total force vector does not lie in the blocks. However by adding a buttress, shown in the above right picture, the buttress provides another force (shown in red) which means the total force now lies inside the arch.



We can even have double flying buttresses, as in the picture above, which add to the aesthetic appeal and can help with other problems, such as unstable ground on which the cathedral is built.

Pinnacles

Another technique that architects employ when building cathedrals is that of a **pinnacle**. Pinnacles are large weights which are added to the top of the arch walls or to the top of the single wall of a buttress. The size of the weight of the pinnacle affects the line of the thrust of the arch.



The pinnacle generally consists of a carved and ornate top piece placed on top of a heavy block weight. At the cathedral in Ely, one pinnacle was never completed as shown in the picture below. However the weight of the base of the pinnacle was large enough to provide the required force so that the arch would still stand.



Since the Middle Ages architects have used flying buttresses and pinnacles to build impressively tall and elegant cathedrals. That these cathedrals have stood for several hundred years is a testament to the builders' profound knowledge of physics and forces.