

A Kaluza-Klein spectrometer from Exceptional Field Theory

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Exceptional Geometry Seminars,
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Based on:

EM, Samtleben: 1911.12640, PRL 124 (2020) 10, 101601

EM, Samtleben: 20xx.xxxx

EM, Nicolai, Samtleben: 20xx.xxxx

The importance of Kaluza-Klein spectra

- Compactification $\Rightarrow$ massive Kaluza-Klein modes arise
- Kaluza-Klein spectrum important
 - AdS/CFT: conformal dimensions
 - Stability of non-SUSY vacua?

Computing Kaluza-Klein spectra is hard

- Free scalar on S^1 :

$$0 = \partial_x^2 \phi(x, y) + \partial_y^2 \phi(x, y),$$

$$\phi(x, y) = \phi^{(k)}(x) e^{i k y / R}, \quad m^2 = \frac{k^2}{R^2}.$$

- SUGRA: (linearised) EoMs mix metric & fluxes $\Rightarrow$ eigenmodes?

$$\nabla_Q f^{QMNP} + \frac{1}{2} F^{QMNP} \nabla_Q h_R{}^R - \nabla_Q (h^{QR} F_R{}^{MNP}) - 3 \nabla^Q (h^{S[M} F_{QS}{}^{NP]}) = -\frac{1}{288} \epsilon^{MNPQ_1 \dots Q_8} F_{Q_1 \dots Q_4} f_{Q_5 \dots Q_8}.$$

- Spin-2 fields [Bachas, Estes '11] ✓
- $M_{int} = \frac{G}{H}$ ✓

... even on G/H

- 11-d on $\text{AdS}_4 \times S^7$ [Englert, Nicolai '83] [Sezgin '84] [Biran, Casher, Englert, Roman, Spindel]
- KK split:

$$g \longrightarrow (g_{ij}, g_{\mu i}, g_{\mu\nu}), \quad C_{(3)} \longrightarrow (C_{ijk}, C_{\mu ij}, \dots),$$

$$C_{(6)} \longrightarrow (C_{ijklmn}, C_{\mu ijklm}, \dots).$$

- Expand in S^7 harmonics

$$g_{ij}(x, y) = \sum_k g^{(k)}(x) \mathcal{Y}_{ij}^{(k)}, \quad C_{ijk}(x, y) = \sum_k C^{(k)}(x) \mathcal{Y}_{ijk}^{(k)},$$

$$g_{\mu i}(x, y) = \sum_k g_{\mu}^{(k)} \mathcal{Y}_i^{(k)}, \quad C_{\mu ij}(x, y) = \sum_k C_{\mu}^{(k)}(x) \mathcal{Y}_{ijk}^{(k)},$$

$$g_{\mu\nu}(x, y) = \sum_k g_{\mu\nu}^{(k)} \mathcal{Y}^{(k)}, \quad \dots$$

- EoMs mix metric & fluxes $\Rightarrow$ diagonalisation problem

BPS multiplets

- BPS multiplets mix different KK levels

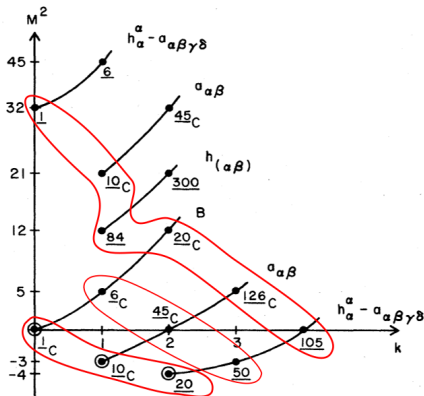


FIG. 2. Mass spectrum of scalars.

[Kim, Romans, Van Nieuwenhuizen '85]

A new method for computing Kaluza-Klein spectra

Our method: [\[Malek, Samtleben 1911.12640\]](#) (for max SUSY AdS_D)

- Only requires scalar harmonics
- Leads to immediate mass diagonalisation
- Organises into supermultiplets
- Can follow deformations by D -dim SUGRA multiplet

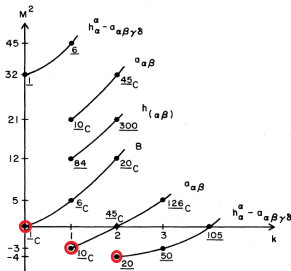
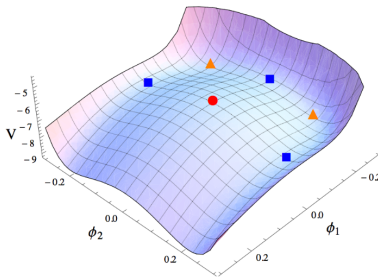


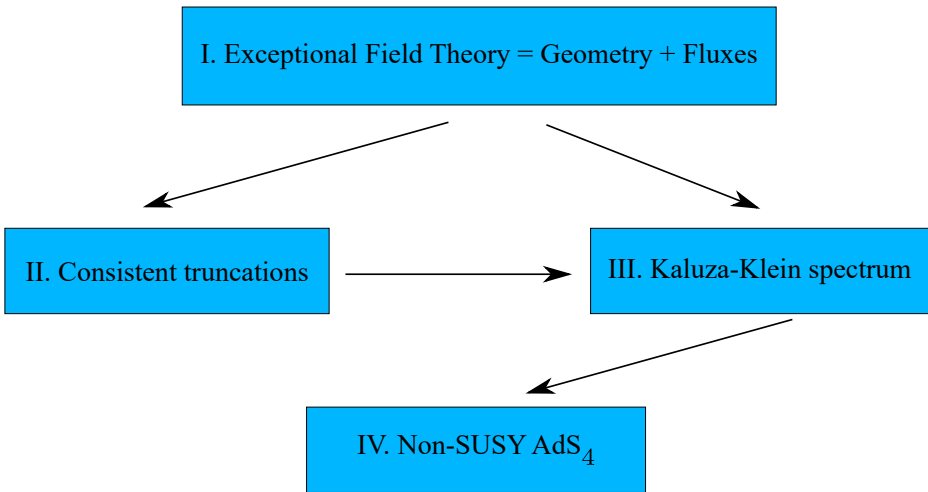
FIG. 2. Mass spectrum of scalars.



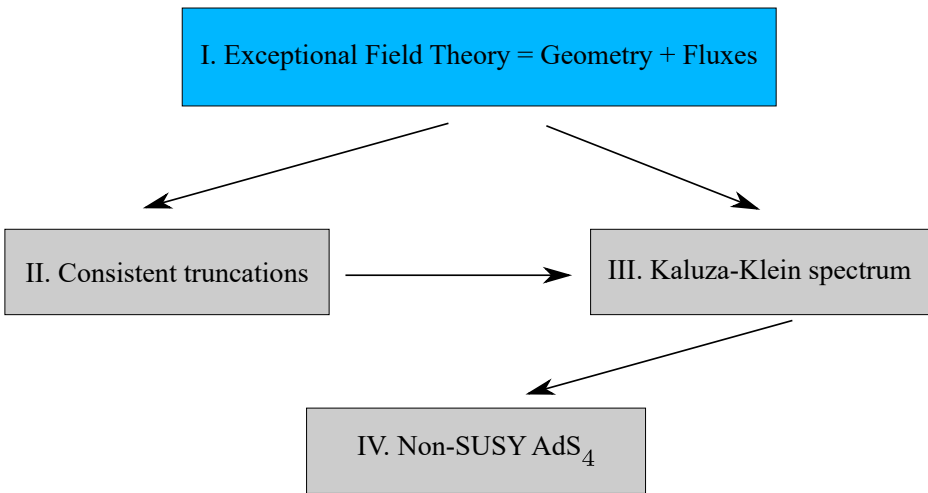
Our methodology

- Mixing of geometry + fluxes $\Rightarrow$ Exceptional Field Theory
- KK Ansatz: $\underbrace{4\text{-D SUGRA multiplet}}_{\text{consistent truncation}} \otimes \text{Scalar harmonics}$
- Cleaner structure
- Non-linear in 4-D $\Rightarrow$ other vacua

Outline



Exceptional Field Theory = Geometry + Fluxes



$E_{7(7)}$ Exceptional field theory

- KK-split of 11-d SUGRA: $(g, C_{(3)}, C_{(6)})$

$$M_{11} = M_4 \times M_{int}.$$

- Unify metric & fluxes:

$$\{g, C_{(3)}, C_{(6)}\} = \mathcal{M}_{MN} \in \frac{E_{7(7)}}{\text{SU}(8)}.$$

- Unify diffeos + gauge symmetries $\rightarrow$ generalised tangent bundle

$$E \simeq TM \oplus \Lambda^2 T^*M \oplus \Lambda^5 T^*M \oplus (T^*M \otimes \Lambda^7 T^*M)$$

$$\Rightarrow E \sim \mathbf{56} \text{ of } E_{7(7)}$$

- Generalised Lie derivative

$$\mathcal{L}_V = V^M \partial_M - (\partial \times_{adj} V) = \text{diffeo} + \text{gauge transf},$$

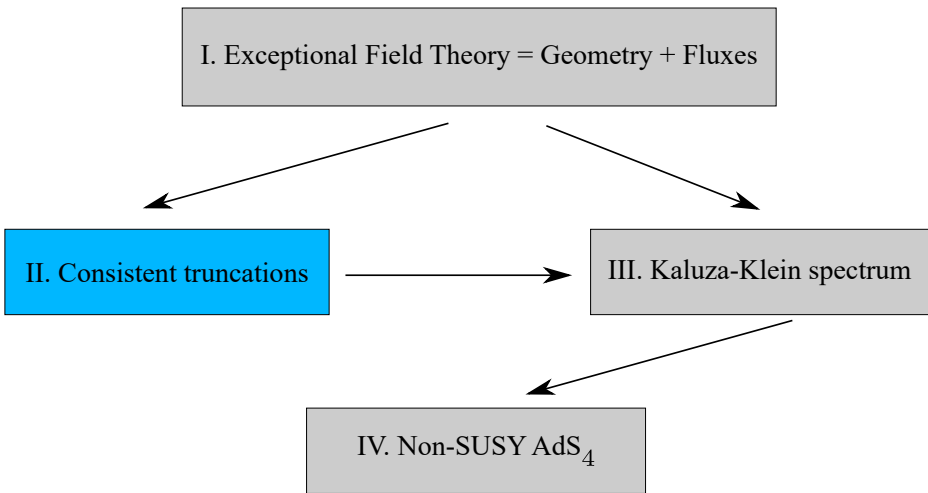
$E_{7(7)}$ ExFT

Full 11-d action: [\[Hohm, Samtleben '13\]](#)

$$L = R_g + \frac{1}{24} g^{\mu\nu} \mathfrak{D}_\mu \mathcal{M}^{MN} \mathfrak{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{F}_{\mu\nu}^M \mathcal{F}^{\mu\nu N} \mathcal{M}_{MN} + L_{top} - V,$$

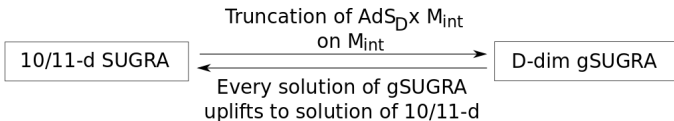
- “Like 4-D SUGRA”
- $\mathfrak{D}_\mu = \partial_\mu - \mathcal{L}_{\mathcal{A}_\mu}$ “external covariant derivatives”
- $\mathcal{F}_{\mu\nu}^M = 2\partial_{[\mu} \mathcal{A}_{\nu]}^M - 2\mathcal{L}_{\mathcal{A}_{[\mu}} \mathcal{A}_{\nu]}^M + \dots$

Consistent truncations from ExFT

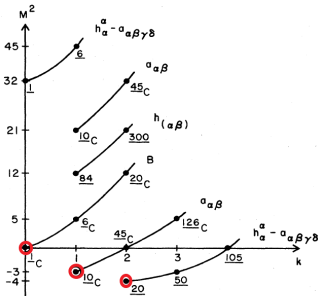


Consistent truncations from AdS vacua

- Maximally SUSY AdS vacua admit consistent truncation



- AdS₄ × S⁷ → 4-D $\mathcal{N} = 8$ SO(8) gSUGRA



Consistent truncations = generalised Scherk-Schwarz

- Generalised Scherk-Schwarz Ansatz: $U_M^A(Y) \in E_{7(7)}$

$$\mathcal{M}_{MN}(x, Y) = M_{AB}(x) U_M^A(Y) U_N^B(Y),$$

$$\mathcal{A}_\mu^M(x, Y) = A_\mu^A(x) U_A^M(Y)$$

- U_A^M generalised frame fields
- Consistency condition:

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C.$$

Proof of consistency

- All Y -dependence in action & EoMs appears via $\mathcal{L}$

$$\mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu}$$

$$\mathcal{F}_{\mu\nu}^M = 2\partial_{[\mu}\mathcal{A}_{\nu]}^M - 2\mathcal{L}_{\mathcal{A}_{[\mu}}\mathcal{A}_{\nu]}^M + \dots$$



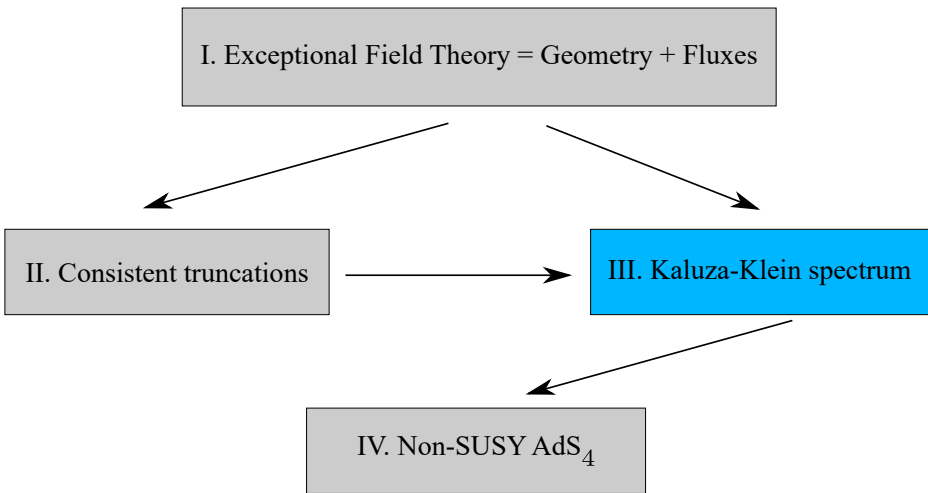
$$\mathcal{M}_{MN}(x, Y) = \mathcal{M}_{AB}(x) U_M^A(Y) U_N^B(Y),$$

$$\mathcal{A}_\mu^M(x, Y) = A_\mu^A(x) U_A^M(Y),$$

$$D_\mu = \partial_\mu - A_\mu^A X_A.$$

$$F_{\mu\nu}^A = 2\partial_{[\mu}A_{\nu]}^A - 2A_\mu^B A_\nu^C X_{BC}^A + \dots$$

Kaluza-Klein spectrum from ExFT



Scalar KK Ansatz at symmetric point

- Background (S^7) described by $U^A_M \in E_{7(7)}$

$$\{g, C_{(3)}, C_{(6)}\} \in \mathcal{M}_{MN} = \delta_{AB} U^A_M U^B_N$$

- Kaluza-Klein Ansatz: linear deformation

$$U^A_M(Y) \longrightarrow \left(\delta^A_B + j^A_B{}^\Sigma(x) \mathcal{Y}_\Sigma \right) U^B_M, \quad j^A_B{}^\Sigma \in \mathfrak{e}_{7(7)}.$$

- Scalar harmonics: $\mathcal{Y}_\Sigma = (1, \mathcal{Y}_a, \mathcal{Y}_{((a}\mathcal{Y}_{b))}, \dots)$, where $\mathcal{Y}_a \mathcal{Y}^a = 1$.

KK Ansatz at symmetric point

- Kaluza-Klein Ansatz: consistent truncation $\otimes$ scalar harmonics

$$\mathcal{M}_{MN}(x, Y) = (\delta_{AB} + \underbrace{2j_{(AB)}{}^\Sigma(x)}_{\in \mathfrak{e}_{7(7)} - \mathfrak{su}(8)} \mathcal{Y}_\Sigma) U^A{}_M(Y) U^B{}_N(Y).$$

$$\mathcal{A}_\mu{}^M(x, Y) = A_\mu{}^A{}^\Sigma(x) U_A{}^M(Y) \mathcal{Y}_\Sigma(Y).$$

- Metric as usual:

$$g_{\mu\nu}(x, Y) = g_{\mu\nu}^0(Y) + h_{\mu\nu}{}^\Sigma(x) \mathcal{Y}_\Sigma.$$

Mass matrices at symmetric point

- Mass matrices

$$\begin{aligned} \square h_{\mu\nu}{}^\Sigma &= (M_g)^\Sigma{}_\Omega h_{\mu\nu}{}^\Omega, & \square j_{AB}{}^\Sigma &= (M_s)_{AB}{}^{\Sigma,CD} j_{CD}{}^\Omega, \\ \square A_\mu{}^{A\Sigma} &= (M_\nu)^{A\Sigma}{}_{B\Omega} A^{B\Omega}, & \dots \end{aligned}$$

- All internal dependence through $\mathcal{L}$

$$\begin{aligned} \mathcal{L}_{U_A} U_B &= X_{AB}{}^C U_C, \\ \mathcal{L}_{U_A} \mathcal{Y}_\Sigma &= L_{\mathcal{K}_A} \mathcal{Y}_\Sigma = \mathcal{T}_{A\Sigma}{}^\Omega \mathcal{Y}_\Omega. \end{aligned}$$

- $\Rightarrow \text{Mass}^2 \sim \mathcal{T}^2, \mathcal{T} \cdot X \text{ and } X^2$

Mass matrices at symmetric point

- Mass matrices:

$$\begin{aligned}
 & M^{AB\Sigma, CD\Omega} j_{AB, \Sigma} j_{CD, \Omega} \\
 &= \frac{1}{7} \left[-6 X^{ACD} X^B{}_{CD} + 2 X^{CAD} X_C{}^B{}_D \right] j_A{}^{E, \Sigma} j_{BE, \Sigma} \\
 &+ \frac{2}{7} \left[2 X^{ACE} X^{BD}{}_E - X^{EAC} X_E{}^{BD} \right] j_{AB}{}^\Sigma j_{CD, \Sigma} \\
 &- 4 X^{ACD} \mathcal{T}^{B, \Omega \Sigma} j_{AB, \Sigma} j_{CD, \Omega} - 4 X^{CAB} \mathcal{T}_C{}^\Omega{}_\Sigma j_A{}^{D, \Sigma} j_{BD, \Omega} \\
 &+ 24 \mathcal{T}_A{}^\Omega{}_\Lambda \mathcal{T}^{B, \Lambda}{}_\Sigma j^{AC, \Sigma} j_{BC, \Omega} - \mathcal{T}^{C, \Omega}{}_\Lambda \mathcal{T}_C{}^\Lambda{}_\Sigma j^{AB, \Sigma} j_{AB, \Omega} ,
 \end{aligned}$$

KK spectrum on S^7

- Break $E_{7(7)} \rightarrow \text{SO}(8)$, $\mathbf{56} \longrightarrow \mathbf{28} \oplus \mathbf{28}$
- $X_{AB}{}^C \rightarrow \text{SO}(8)$
- Harmonics $\mathcal{Y}_\Sigma = (1, \mathcal{Y}_a, \mathcal{Y}_{ab}, \dots) \in [n, 0, 0, 0]$

$$\mathcal{T}_A{}^\Sigma{}_\Omega = \left(\underbrace{\mathcal{T}_{ab}{}^{e_1 \dots e_n}{}_{f_1 \dots f_n}}_{\text{SO}(8) \text{ generators}}, \underbrace{\mathcal{T}^{ab, e_1 \dots e_n}{}_{f_1 \dots f_n}}_{=0} \right).$$

$$j^A{}_{B\Sigma} = (\phi_{ab}, \phi_{abcd})_{,e_1 \dots e_n}.$$

S^7 mass diagonalisation and BPS multiplets

$$\phi_{ab,e_1\dots e_n} \ni [n+2, 0, 0, 0] \oplus [n-2, 2, 0, 0] \oplus \dots$$

- Each irrep $\rightarrow$ 1 mass eigenstate
- $[n, 0, 0, 0] \rightarrow$ 1 BPS multiplet
- Mixing from $U_A^M \mathcal{Y}_\Sigma$ and ExFT $\leftrightarrow$ 11-d dictionary

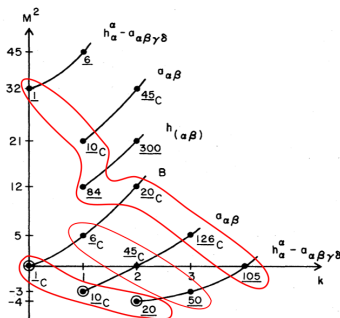
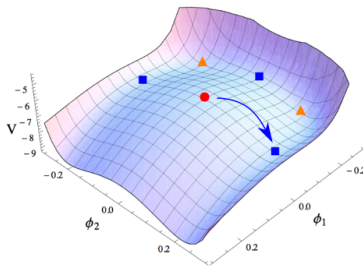


FIG. 2. Mass spectrum of scalars.

Moving to a less symmetric vacuum

- Vacuum $M_{AB}(x) = \mathcal{V}^{\bar{A}}{}_{A}(x) \mathcal{V}^{\bar{B}}{}_{B}(x) \delta_{\bar{A}\bar{B}}$
- $U^A{}_M \longrightarrow U^{\bar{A}}{}_M = \mathcal{V}^{\bar{A}}{}_B U^B{}_M.$

$$\mathcal{L}_{U_{\bar{A}}} U_{\bar{B}} = X_{\bar{A}\bar{B}}{}^{\bar{C}} U_{\bar{C}}, \quad X_{\bar{A}\bar{B}}{}^{\bar{C}} = \mathcal{V}_{\bar{A}}{}^A \mathcal{V}_{\bar{B}}{}^B \mathcal{V}^{\bar{C}}{}_C X_{AB}{}^C.$$



Harmonics at less symmetric vacuum

- Use same harmonics $\mathcal{Y}_\Sigma$ as max. symmetric vacuum.
- Linear action on harmonics:

$$\mathcal{L}_{U_{\bar{A}}} \mathcal{Y}_\Sigma = L_{\mathcal{K}_{\bar{A}}} \mathcal{Y}_\Sigma = \mathcal{T}_{\bar{A}\Sigma}{}^\Omega \mathcal{Y}_\Omega,$$

$$\mathcal{T}_{\bar{A}\Sigma}{}^\Omega = \mathcal{V}_{\bar{A}}{}^A \mathcal{T}_{A\Sigma}{}^\Omega,$$

Kaluza-Klein masses at less symmetric vacuum

- Harmonic analysis *at max. symmetric point*:
 - Scalar harmonics
 - $\mathcal{T}_{A,\Omega}{}^\Sigma$
- All deformation info from “dressing”:

$$\begin{aligned}
 X_{AB}{}^C &\longrightarrow X_{\bar{A}\bar{B}}{}^{\bar{C}}, & \mathcal{T}_{A\Omega}{}^\Sigma &\longrightarrow \mathcal{T}_{\bar{A},\Omega}{}^\Sigma, \\
 X_{\bar{A}\bar{B}}{}^{\bar{C}} &= \mathcal{V}_{\bar{A}}{}^A \mathcal{V}_{\bar{B}}{}^B \mathcal{V}^{\bar{C}}{}_C X_{AB}{}^C, & \mathcal{T}_{\bar{A},\Omega}{}^\Sigma &= \mathcal{V}_{\bar{A}}{}^A \mathcal{T}_{A\Omega}{}^\Sigma.
 \end{aligned}$$

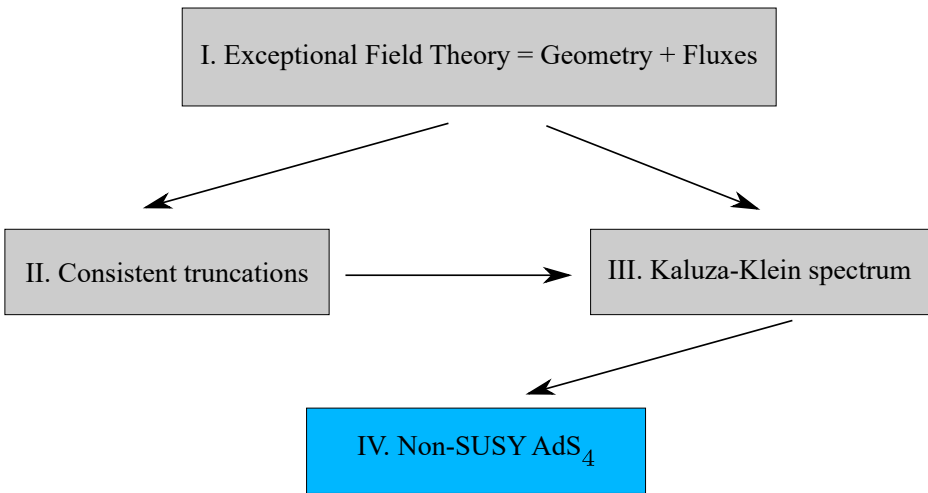
Kaluza-Klein masses at less symmetric vacuum

- Harmonic analysis *at max. symmetric point*:
 - Scalar harmonics
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 X_{AB}{}^C &\longrightarrow X_{\bar{A}\bar{B}}{}^{\bar{C}}, & \mathcal{T}_{A\Omega}{}^\Sigma &\longrightarrow \mathcal{T}_{\bar{A},\Omega}{}^\Sigma, \\
 X_{\bar{A}\bar{B}}{}^{\bar{C}} &= \mathcal{V}_{\bar{A}}{}^A \mathcal{V}_{\bar{B}}{}^B \mathcal{V}^{\bar{C}}{}_C X_{AB}{}^C, & \mathcal{T}_{\bar{A},\Omega}{}^\Sigma &= \mathcal{V}_{\bar{A}}{}^A \mathcal{T}_{A\Omega}{}^\Sigma.
 \end{aligned}$$

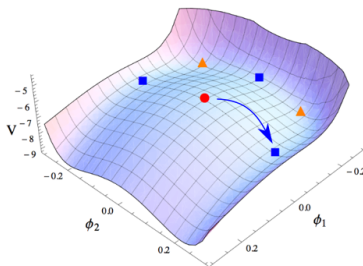
$$\begin{aligned}
 M^{\bar{A}\bar{B}\Sigma, \bar{C}\bar{D}\Omega} j_{\bar{A}\bar{B},\Sigma} j_{\bar{C}\bar{D},\Omega} &= \frac{1}{7} \left[-6 X^{\bar{A}\bar{C}\bar{D}} X^{\bar{B}}{}_{\bar{C}\bar{D}} + 2 X^{\bar{C}\bar{A}\bar{D}} X_{\bar{C}}{}^{\bar{B}}{}_{\bar{D}} \right] j_{\bar{A}}{}^{\bar{E},\Sigma} j_{\bar{B}\bar{E},\Sigma} \\
 &+ \frac{2}{7} \left[2 X^{\bar{A}\bar{C}\bar{E}} X^{\bar{B}\bar{D}}{}_{\bar{E}} - X^{\bar{E}\bar{A}\bar{C}} X_{\bar{E}}{}^{\bar{B}\bar{D}} \right] j_{\bar{A}\bar{B}}{}^\Sigma j_{\bar{C}\bar{D},\Sigma} \\
 &- 4 X^{\bar{A}\bar{C}\bar{D}} \mathcal{T}^{\bar{B},\Omega\Sigma} j_{\bar{A}\bar{B},\Sigma} j_{\bar{C}\bar{D},\Omega} - 4 X^{\bar{C}\bar{A}\bar{B}} \mathcal{T}_{\bar{C}}{}^\Omega{}_\Sigma j_{\bar{A}}{}^{\bar{D},\Sigma} j_{\bar{B}\bar{D},\Omega} \\
 &+ 24 \mathcal{T}_{\bar{A}}{}^\Omega{}_\Lambda \mathcal{T}^{\bar{B},\Lambda}{}_\Sigma j^{\bar{A}\bar{C},\Sigma} j_{\bar{B}\bar{C},\Omega} - \mathcal{T}^{\bar{C},\Omega}{}_\Lambda \mathcal{T}_{\bar{C}}{}^\Lambda{}_\Sigma j^{\bar{A}\bar{B},\Sigma} j_{\bar{A}\bar{B},\Omega},
 \end{aligned}$$

Non-SUSY AdS₄



Non-supersymmetric AdS₄

- 4-D SO(8) gSUGRA $\supset$ SO(3) $\times$ SO(3)-invariant $\mathcal{N} = 0$ AdS₄ vacuum
[Warner '83]



- Uplift to 11-d [Godazgar, Godazgar, Kruger, Nicolai, Pilch]
- Fully stable in 4-D! [Fishbacher, Pilch, Warner '10]
Only known example in 4-D.

Stability in 11-d?

- Uplift to 11-d stable? Higher KK modes?
- AdS swampland conjecture [Ooguri, Vafa '16]
- Brane-Jet instability [Bena, Pilch, Warner '20]

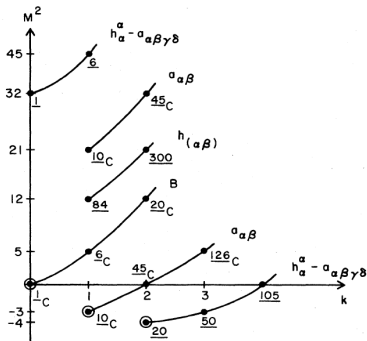
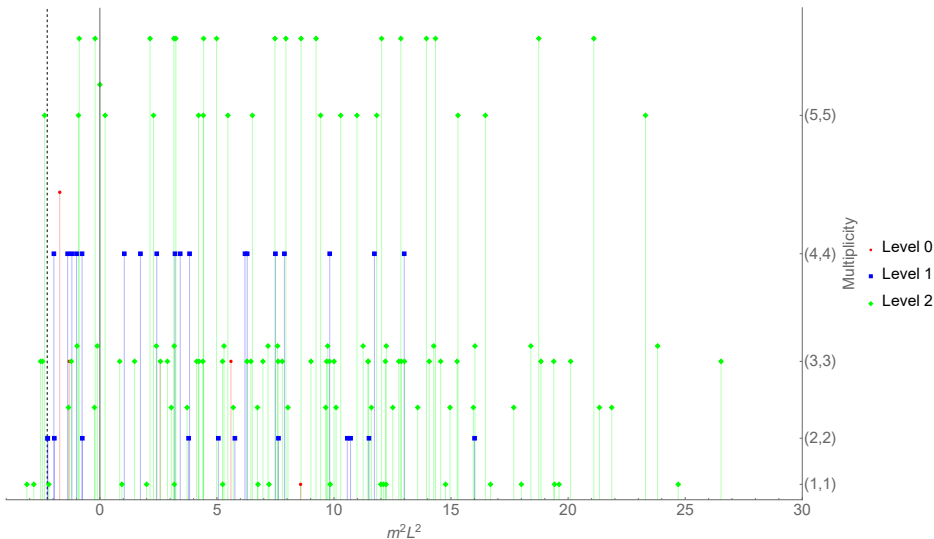


FIG. 2. Mass spectrum of scalars.

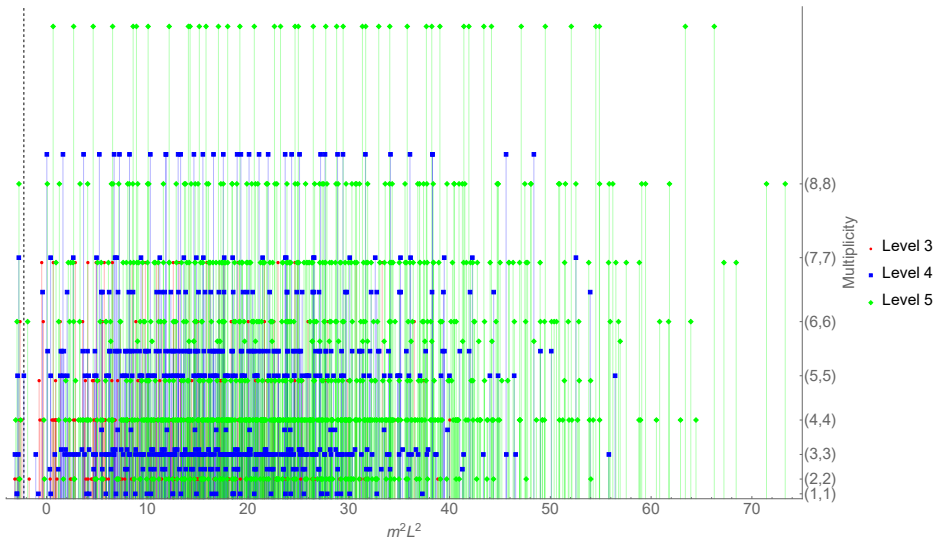
Non-supersymmetric KK spectroscopy

- Harmonic analysis for round S^7 (SO(8) point)
 - Scalar harmonics $\mathcal{Y}^\Sigma = (1, \mathcal{Y}^a, \mathcal{Y}^{ab}, \dots, \mathcal{Y}^{a_1 \dots a_n}, \dots)$
 - $\mathcal{T}_{A,\Omega}^\Sigma \rightarrow \mathcal{T}_{A,a_1 \dots a_n}{}^{b_1 \dots b_n} \rightarrow \text{SO}(8) \text{ generators}$
- Lower-dimensional info captures deformation
 - $\mathcal{V}_{\bar{A}}^A$ from [Warner '83].
 - $\rightarrow$ dressed $X_{\bar{A}\bar{B}}{}^{\bar{C}}, \mathcal{T}_{\bar{A},\Omega}^\Sigma$.

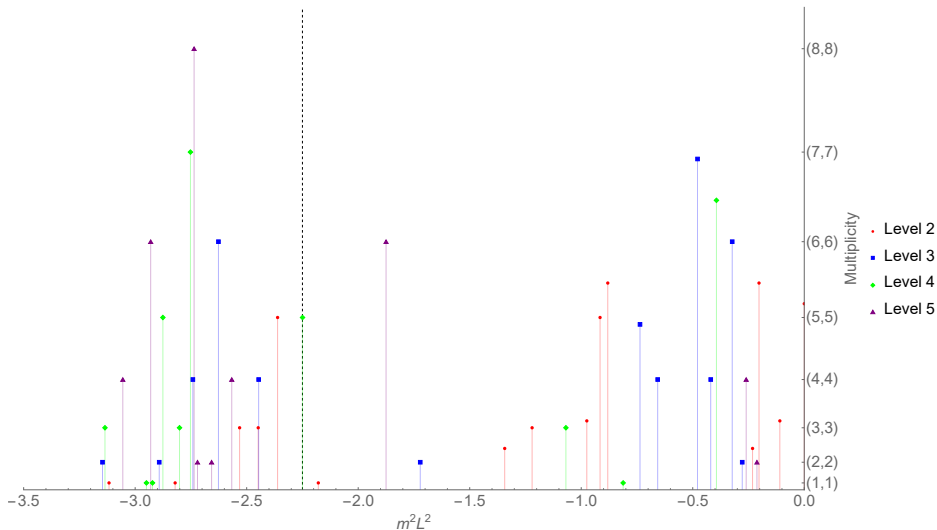
KK spectrum levels 0 – 2



KK spectrum levels 3 – 5



KK Tachyons



Tachyonic Kaluza-Klein modes

$$\text{Level 2: } 2 \cdot (\mathbf{1}, \mathbf{1}) \oplus 2 \cdot (\mathbf{3}, \mathbf{3}) \oplus (\mathbf{5}, \mathbf{5}),$$

$$\text{Level 3: } 2 \cdot (\mathbf{2}, \mathbf{2}) \oplus 2 \cdot (\mathbf{4}, \mathbf{4}) \oplus (\mathbf{6}, \mathbf{6}),$$

$$\text{Level 4: } 2 \cdot (\mathbf{1}, \mathbf{1}) \oplus 2 \cdot (\mathbf{3}, \mathbf{3}) \oplus (\mathbf{5}, \mathbf{5}) \oplus (\mathbf{7}, \mathbf{7}),$$

$$\text{Level 5: } 2 \cdot (\mathbf{2}, \mathbf{2}) \oplus 2 \cdot (\mathbf{4}, \mathbf{4}) \oplus (\mathbf{6}, \mathbf{6}) \oplus (\mathbf{8}, \mathbf{8}).$$

Moduli

- 27 massless scalar fields at level 2
- $3 \cdot (\mathbf{3}, \mathbf{3}) \Rightarrow$ break $SO(3) \times SO(3)$ symmetry!
- Finite deformations?
- Family of non-SUSY AdS₄ vacua of 11-d SUGRA?

General procedure

- Given consistent truncation to $\mathcal{N} = 8$ D -dim SUGRA
- Scalar harmonics on $M_{int} = G/H$ at maximally symmetric point
- Can compute KK masses for *any* vacuum of D -dim SUGRA

AdS₄ vacua of ISO(7) SUGRA

- Consistent truncation of massive IIA on $S^6 \Rightarrow \mathcal{N} = 8$ ISO(7) gSUGRA
[Guarino, Jafferis, Varela '15]
 - $\mathcal{N} = 2$ AdS₄ [Guarino, Jafferis, Varela '15]
 - $\mathcal{N} = 3$ AdS₄ vacua [Gallerati, Samtleben, Trigiante '14]
- No maximally supersymmetric vacuum
- Round S^6 not a solution!
- But use harmonics on S^6 for KK spectrum

Conclusions

- Powerful method for computing KK spectra
- KK spectrum of less symmetric vacua
- Non-SUSY $\text{SO}(3) \times \text{SO}(3)$ AdS_4 vacuum is unstable
c.f. Brane-Jet instability [Bena, Pilch, Warner '20]
- Existence of consistent truncation has bigger implications!

Outlook

- Non-SUSY AdS₄
 - End-point of instability?
 - Finite moduli?
 - Other examples in 3-D [[Fischbacher '02](#)] [[Fischbacher, Nicolai, Samtleben '02](#)]
- KK spectrum of SUSY AdS vacua
 - Compute indices (c.f. [[Ashmore, Petrini, Tasker, Waldram](#)])
 - Understanding of long multiplets
 - Correlation functions
- Extension to consistent truncations with less SUSY
⇒ KK spectrum of 1/2-max AdS vacua, c.f. spin-2 [[Chen, Gutperle, Uhlemann '19](#)], [[Gutperle, Uhlemann, Varela '18](#)]