Topology and geometry for physicists

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Abstract

These lecture notes are based on a series of lectures given in 2017 at the XIII Modave Summer School in Mathematical Physics. The lectures are aimed at beginning PhD students in theoretical physics and cover topics in homotopy theory, homology and cohomology, as well as fibre bundles, with applications in condensed matter systems, electromagnetism and Yang-Mills gauge theories.

Note to the reader

These lecture notes are based on a five hour lecture course given at the XIII Modave Summer School in Mathematical Physics. The course was aimed at beginning PhD students in theoretical physics and aim to introduce some of the important concepts in topology and geometry, in particular homotopy theory, homology and cohomology and fibre bundles, which the students are likely to encounter during their careers. Applications of these concepts are discussed at the end of the respective sections. Due to the shortness of the course and its target audience, the emphasis in these notes is not on mathematical rigour but instead on conveying the important concepts in a way that is hopefully intuitive to physicists. Therefore, proofs of theorems are only given when they are instructive.

On the other hand, the notes include exercises interspersed throughout the text. Any reader who wants to gain more than a superficial understanding of the material, should attempt at least a large proportion of the exercises. Particularly difficult exercises are marked with an asterik.

These notes assume a basic knowledge of topology and differential manifolds, to the standard introduced in a typical Master's course on general relativity. There are several excellent books where more details can be found that were not covered in these notes. In particular, I can recommend the books "Geometry, topology and physics" by Nakahara, as well as "Geometry and topology for physicists" by Nash & Sen. For the more mathematically minded readers, the book "Algebraic Topology" by Hatcher is a fantastic resource, available for download for *free* at `https://www.math.cornell.edu/~hatcher/AT/ATpage.html`. There are also many very good online resources, for example the lecture notes of the Edinburgh Mathematical Physics Group postgraduate course on "Gauge theories" which are available at `https://empg.maths.ed.ac.uk/Activities/GT/`.