Quantum Field Theory: Example Sheet 3

Prof. M. J. Perry, 22 November 2013

1. Starting from the Maxwell action, find the wave equation in the Coulomb gauge,

$$\nabla \cdot \mathbf{A} = 0.$$

What is the equation that determines A_0? Show that it is sensible to fix $A_0 = 0$. Is the operator P_{ij} are a projection operator when

$$P_{ij} = \delta_{ij} - \Big(\frac{\nabla_i \nabla_j}{\nabla^2}\Big).$$

Construct the Fourier transform of the propagator function in this gauge. Why does this give the same answers for physical processes at tree level as the propagator found in lectures?

2. A non-relativistic one-dimensional simple harmonic oscillator of angular frequency ω and mass m has Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

where p is the momentum operator and x the position operator. Use the path integral method to find the amplitude for the transition from position x_i and time t_i to x_f and time t_f.

3. Show that the Feynman rules provided for scalar fields in the lectures follow from the path integral. An outline of this calculation can be found in most books on Quantum Field Theory, e.g. Zee pp $49 - 55$.

4. Suppose that one has an interaction of the form $\frac{\lambda}{3!}\phi^3$. Calculate the amplitude for two particles scattering into two particles in the tree-level approximation. What is the differential cross-section?

Please address any comments, especially about errors and omissions to: mjp1@cam.ac.uk