

Introduction to String Theory

Lecture 1

Introduction

- only some topics mostly bosonic closed string theory
- intro to AdS/CFT (Forini) SoSe
- self-learning

String theory

- Quantum theory of fundamental strings
- ↓
- Quantum Gravity

Why this course?

- ① Quantum Gravity
- Big Bang (still contentious)
 - Black Holes (already successful)
e.g. entropy

- ② Ideas for QG
- Multiverse
 - Extra dimensions
 - Swampland (hot topic)
 - Dualities

- ③ Tool for gauge theories
- AdS/CFT
 - CFT
 - D-branes

- ④ Mathematics
- Diff. Geometry
 - Mirror Symmetry
 - Higher gauge theories

Features of string theory:

- 2-d QFT
- GR is not built in but arises naturally
- 2 kinds of string theory: closed & open

- Intricate non-perturbative sector: D-branes
dualities
M-theory
- SUSY is crucial but not in this course

Prerequisites:

QFT I } at the very least !!!
GR } in parallel with this ...
course

Lectures Fridays 9am - 11am
 or
 11am - 1pm

& alt. Mondays 9am-11am, starting on 16th Nov
 Tutorials alt. Mondays 9am-11am, starting on 9th Nov

- Outline:
- Classical bosonic string
 - Light-cone quantisation
 - D-branes
 - Conformal Field Theory
 - Path integral quantisation
 - String interactions
 - Low-energy effective actions
 - Superstring
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① Classical bosonic string

I - Relativistic point particle

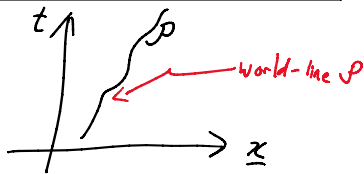
Minkowski space $M = \mathbb{R}^{D-1} = (M, \gamma)$

Space-time signature $(-, \underbrace{+, \dots, +}_{D-1 \text{ times}})$

$$\hbar = c = 1$$

Point particle:

$$X^\mu = (t, \underline{x})$$



First version

Action: $S_{(0)} = -m \int_{\mathcal{P}} dt \sqrt{1 - \dot{\underline{x}} \cdot \dot{\underline{x}}}, \quad \dot{\underline{x}} = \frac{d}{dt} \underline{x}$

Dynamical DoF: $\underline{x}(t) \quad (D-1 \text{ DoF's})$

Correct, since: $\underline{p} = \frac{\delta S_{(0)}}{\delta \dot{\underline{x}}} = \frac{m \dot{\underline{x}}}{\sqrt{1 - \dot{\underline{x}} \cdot \dot{\underline{x}}}}$ } *qs required*

$\& \xrightarrow{\text{Hamiltonian}} E = \sqrt{m^2 + \underline{p} \cdot \underline{p}} = \frac{m}{\sqrt{1 - \dot{\underline{x}} \cdot \dot{\underline{x}}}}$

EoM: $\dot{\underline{p}} = 0$

Symmetries: Only spatial rotations & translations (global)
 $\underline{x} \rightarrow R \underline{x} + \underline{s}$

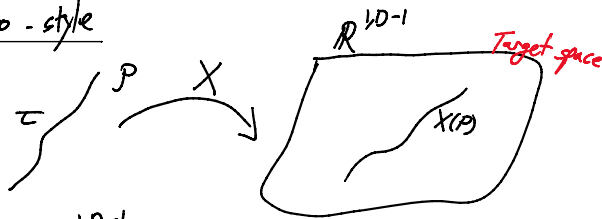
Features: NOT Lorentz invariance! $\underline{x}$ & t are treated differently!
 $\uparrow$ dynamical
 $\uparrow$ parameter

1.1, do better!

dynamical parameter

Let's do better!

Nambu-Goto-style



$$X: P \rightarrow \mathbb{R}^{D-1}$$

i.e. $X(\tau) \in \mathbb{R}^{D-1} \rightarrow$ w/ local coordinates $X^\mu(\tau)$

$\mu = 0, \dots, D-1$

Dynamical DoF: $X^\mu(\tau)$ (D DoF's)

Action: Spec. Rel. $\Rightarrow$ minimise proper time $\int ds$
w/ $ds^2 = -\dot{X}^\mu \dot{X}^\nu g_{\mu\nu} d\tau^2$

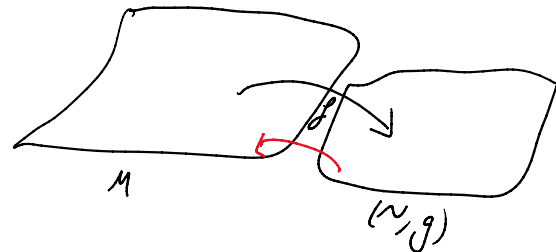
$$S_{NG} = -m \int_P d\tau \sqrt{-\dot{X}^\mu \dot{X}^\nu g_{\mu\nu}} = -\int_P ds$$

$$= -m \int_P d\tau \sqrt{-\det(X^* \eta)}$$

$$\det(1-\alpha) = (1-\alpha)$$

Pull-back
Exercise Sheet 1
Problem 2

$$\dot{X}^\mu = \frac{dX^\mu(\tau)}{d\tau}$$



Pull-back Metric on M : f^*g

Local coords σ^α on M
 X^μ on N

Symmetries: (i) 1-dim diffeos $\rightarrow$ reparametrisations of τ (Local)

$$\tau \rightarrow \tau'(\tau)$$

Scalars $X^\mu(\tau) \rightarrow X^\mu(\tau')$

Show that:

$$S' = -m \int_{P'} d\tau' \sqrt{-\frac{dX^\mu}{d\tau'} \frac{dX^\nu}{d\tau'} g_{\mu\nu}}$$

$$S = -m \int_P d\tau \sqrt{-\frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} g_{\mu\nu}}$$

$$S_{NG} = S'_{NG} \text{ is invariant!}$$

(ii) Poincaré symmetry (Global)

$$X^\mu(\tau) \rightarrow \Lambda^\mu_\nu X^\nu(\tau) + C^\mu$$

w/ $\Lambda_{\mu\nu} = 0 \Rightarrow$ Lorentz transf. + translation

EoM:

$$P_\mu = \frac{\delta S}{\delta \dot{X}^\mu} = \frac{m \dot{X}^\mu}{\sqrt{-\dot{X}^\nu \dot{X}^\mu g_{\nu\mu}}}$$

NB: Only $D-1$ P_μ 's are independent!

$$P_\mu P^\mu + m^2 = 0 \text{ - Mass-shell condition!}$$

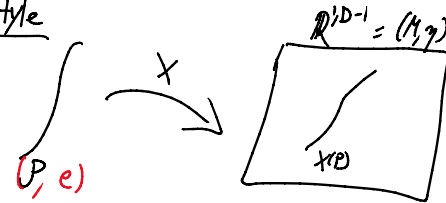
$\rightarrow$ Another sign that we have a gauge symmetry

$$h_{\alpha\beta} = (f^*g)_{\alpha\beta} = \frac{\partial f^\mu}{\partial \sigma^\alpha} \frac{\partial f^\nu}{\partial \sigma^\beta} g_{\mu\nu}$$

$|\mu| + m \rightarrow \dots$
 $\rightarrow$ Another sign that we have a gauge symmetry
 $\rightarrow (p^0)^2 \geq m^2 \Rightarrow$ Particle has to move in time!

Gauge fixing: Choose $\tau = X^0(\tau) \equiv t \leftarrow$ Breaks Poincaré symmetry
 $\Rightarrow S_{NG} \rightarrow S_{(t)} = -m \int dt \sqrt{1 - \dot{x}^2}, \quad \dot{x} = \frac{dx}{dt}$
 Gauge choice breaks Poincaré $\Rightarrow S_{(t)}$ has no Poincaré symmetry

Features: We have restored Poincaré at the "cost" of introducing extra gauge symmetry.

Polyakov-style

 $X: \mathcal{P} \rightarrow \mathbb{R}^{D-1}$

But introduce 1-d gravity to get rid of $\sqrt{\dots}$

Dynamical DoF: $X^\mu(\tau)$ & $e(\tau) \leftarrow$ 1-d gravity
 $D+1$ DoF's

Action: $S_{(P)} = \frac{1}{2} \int_{\mathcal{P}} d\tau (\bar{e}^{-1} \dot{X}^2 - e m^2) \quad \Big| \quad \dot{X}^2 = \dot{X}^\mu \dot{X}^\nu g_{\mu\nu}$

NB: write $e = \sqrt{-g_{\tau\tau}} = \sqrt{-\det g}$
 $\Rightarrow S_{(P)} = \frac{1}{2} \int_{\mathcal{P}} d\tau \sqrt{-g_{\tau\tau}} (g^{\tau\tau} \dot{X}^2 - m^2) \quad \Big| \quad g_{\tau\tau} = \frac{1}{g^{\tau\tau}}$

Symmetries: (i) 1-d diffeos (Local)

$\tau \rightarrow \tau'(\tau)$
 Scalars $X^\mu(\tau') = X^\mu(\tau)$
 Scalar density $e'(\tau') = e(\tau) \left| \frac{d\tau}{d\tau'} \right|$

Exercise Sheet 1
 Problems 1 & 4

(ii) Poincaré Symmetry (Global)

$X^\mu(\tau) \rightarrow \Lambda^\mu_\nu X^\nu(\tau) + C^\mu$ w/ $\Lambda_{\mu\nu} = 0 \quad \Big| \quad \Lambda_{\mu\nu} = \frac{1}{2}(\Lambda_{\mu\nu} + \Lambda_{\nu\mu})$

EoMs: NB: e only appears algebraically

$\frac{\delta S_P}{\delta e} = 0 \Rightarrow T = \dot{X}^2 + e^2 m^2 = 0$

Exercise Sheet 1
 Problem 3

$\Rightarrow e = \sqrt{-\frac{\dot{X}^2}{m^2}}$ is fixed by EoM's
 $\& e \sim \sqrt{\det(-\eta_{\mu\nu})}$
 $\dot{X}^2 = -\dot{X}^2$

$\frac{\delta S_P}{\delta X^\mu} = 0 \Rightarrow \frac{d}{d\tau} (e^{-1} \dot{X}^\mu) = 0$

NB: Plug $T=0 \Rightarrow e = \sqrt{-\frac{\dot{X}^2}{m^2}}$ into $S_P \rightarrow S_{NG}$

Exercise Sheet 1
 Problem 4

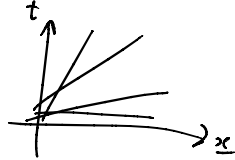
Gauge fixing: Instead of solving for e & recovering S_{NG} ,
 we can instead use local diffeos to fix

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$$\rightarrow e(\tau) = \frac{1}{m}$$

$\Rightarrow$ Gauge-fixed EoMs: $\ddot{X}^\mu = 0$

still have $T=0$ & $\dot{X}^2 = -1$ (from $T=0$)



! NB: we must still impose $T=0$ to ensure we extremise the action !

Features:

- Added 1-d gravity $\rightarrow$ got rid of $\sqrt{\quad}$
- Extra DoF $e(\tau)$ appeared algebraically!

$\Rightarrow$ Can "integrate out" $e(\tau)$ or gauge fix it

$\swarrow$
 S_{NG}

$\searrow$ simple EoM's
+ constraint $T=0$

- Aside: S_p also defines a point particle action for $m \neq 0$

Exercise: Find EoMs for S_p w/ $m \neq 0$
Gauge fix $e(\tau)$ & write down EoMs
& constraints $T \equiv \frac{\delta S_p}{\delta e} = 0$