Lecture 19

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Recap:

- Strings in background fields $S \to S_{G,B,\phi}$ is invariant under spacetime gauge symmetries:
  - diffeos
  - B-field gauge transformations
- $\langle \phi \rangle \Leftrightarrow$ sets $g_s \Leftrightarrow$ no free dimensionless parameters in string theory
- Weyl invariance in curved spacetime $\Rightarrow$ β-function must vanish
- At 1-loop: $\mathcal{R}_{\mu\nu} = 0 \quad (\text{w/ } B = \phi = 0)$
- Corrections to Einstein equations:

  higher loop $\Rightarrow \quad \mathcal{R}_{\mu\nu} + \frac{\alpha'}{2} \mathcal{R}_{\mu\lambda\rho\sigma} \mathcal{R}_{\nu}{}^{\lambda\rho\sigma} = 0$
- LEEA for $\beta(G) = \beta(B) = \beta(\phi) = 0$ as EoM's

$$\Rightarrow S_{LEEA} = \frac{1}{2\kappa^2} \int d^{26}X \sqrt{-G}\, e^{-2\phi} \left(\mathcal{R} - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + 4 \partial_\mu \phi \partial^\mu \phi \right)$$

- 2 perturbative expansions in string theory,
  - g_s in string loop $\Leftrightarrow$ "spacetime perturbative expansion"
  - α' in worldsheet perturbation theory

    because w/ $G \neq \eta$, $B \neq 0$, $\phi = 0$, worldsheet is not free!

NB: string scattering amplitudes give g_s corrections to LEEA from loop diagrams. These also appear in β-function calculation! g_s effects come from higher

β-function calculation! g_s effects come from higher genus worldsheets $\Rightarrow$ different topologies

$\Rightarrow$ affect propagator $\langle Y^\mu(\sigma) Y^\nu(\sigma')\rangle$

However, some points in moduli space correspond to degenerating Riemann surfaces $\Rightarrow$ also modified UV behaviour! $\Rightarrow$ g_s corrections to LEEA.

(vii) Einstein vs String frame

$$S_{LEEA} = \frac{1}{2\kappa^2}\int d^{26}X \sqrt{-G}\, e^{-2\phi}\left(R - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + 4\partial_\mu\phi\partial^\mu\phi\right)$$

not quite conventional Einstein-Hilbert term

kinetic term seems to have wrong sign

The action looks unconventional due to $e^{-2\phi}$ out front & "bad" kinetic term for ϕ.

related to fact that this comes from tree level $\Rightarrow g_s^{-2}$

We can perform a Weyl rescaling of spacetime metric G to absorb $e^{-2\phi}$:

$$G_{\mu\nu} \to \tilde{G}_{\mu\nu} = e^{2\omega} G_{\mu\nu}$$

$$\Rightarrow \tilde{R} = e^{-2\omega}\left(R - 2(D-1)\nabla^2\omega - (D-2)(D-1)\partial_\mu\omega\partial^\mu\omega\right)$$

taking into account $\sqrt{G}$ factor, we need to take

Taking into account $\sqrt{G}$ factor, we need to take

$$\omega = -\frac{2\phi}{D-2} = -\frac{\phi}{12}$$

$$\Rightarrow S_{LEEA} = \frac{1}{2\kappa^2}\int d^{26}X \underbrace{\sqrt{-\tilde{G}}\,\big(\tilde{R}}_{\text{E-H!}} - \frac{1}{12}e^{-\tilde{\phi}/3} H_{\mu\nu\lambda}H^{\mu\nu\lambda} \underbrace{- \frac{1}{6}\partial_\mu\phi\,\partial^\mu\phi}_{\substack{\text{right sign} \\ \text{in kinetic} \\ \text{term!}}}\big)$$

Since $\tilde{G}$ has canonical Einstein-Hilbert term, it is known as

Einstein frame metric

Since G couples to the string worldsheet action, it is known as

String frame metric

More generally, whenever we have a massless scalar field coupled to GR, we have this ambiguity of redefining the metric & coupling matter fields to redefined metrics

These different couplings violate equivalence principle
$\Rightarrow$ give rise to unobserved "fifth forces"

$\Updownarrow$

When trying to model our universe we should find a way to make all scalar fields massive in string theory ("moduli stabilisation")

Not the moduli of Riemann

Not the moduli of Riemann surfaces from scattering amplitudes

NB: In cosmology, the analogue of string frame is called

Jordan frame.

III - Superstring low-energy effective actions

Let's have a brief look at what happens in superstring.

Massless excitations of closed string are:

Common sector "Neveu-Schwarz" NS		Extra bosons "Ramond-Ramond" RR		Fermionic excitations	
$G_{\mu\nu}$, $B_{\mu\nu} = B_{(2)}$, ϕ	$\oplus$	$C_{(1)}, C_{(3)}$	$\oplus$	Fermions	IIA superstring
		$C_{(0)}, C_{(2)}, C_{(4)}$		Fermions	IIB superstring
		$A_{(1)}$		Fermions	Heterotic superstring

For each superstring, we obtain a low-energy effective action:

$$S_{LEEA} = S_{NS} + S_{RR} + S_{fermi}$$

(i) $$S_{NS} = \frac{1}{2\kappa^2}\int d^{10}X \sqrt{-G}\, e^{-2\phi}\left(R - \frac{1}{2}|H_{(3)}|^2 + 4\partial_\mu\phi\,\partial^\mu\phi\right)$$

(i) $S_{NS} = \frac{1}{2\kappa^2}\int d^{10}x \sqrt{-G}\, e^{-2\Phi}\left(R - \frac{1}{2}|H_{(3)}|^2 + 4 g_{\mu\nu}\partial^\mu\phi\,\partial^\nu\phi\right)$

$$\underbrace{H_{(3)} = dB_{(2)}}_{\text{not quite true in Het string!}} \quad \& \quad |H_{(3)}|^2 = \frac{1}{3!} H_{\mu\nu\rho} H^{\mu\nu\rho}$$

(ii) <u>IIA</u>

$$S_{RR} = -\frac{1}{4\kappa^2}\int d^{10}x \Big[\sqrt{-G}\left(|F_{(2)}|^2 + |\tilde{F}_{(4)}|^2\right) + \underbrace{B_{(2)}\wedge F_{(4)}\wedge F_{(4)}}_{\text{"topological term" does not depend on } G \;\leftrightarrow\; \text{Chern-Simons Terms}}\Big]$$

$F_{(2)} = dC_{(1)}$, $F_{(4)} = dC_{(3)}$

$\tilde{F}_{(4)} = F_{(4)} - C_1 \wedge H_3$

<u>IIB</u>

$$S_{RR} = -\frac{1}{4\kappa^2}\int d^{10}x \Big[\sqrt{-G}\left(|F_{(1)}|^2 + |\tilde{F}_{(3)}|^2 + \frac{1}{2}|\tilde{F}_{(5)}|^2\right) + C_4 \wedge H_3 \wedge F_3\Big]$$

$F_{(p)} = dC_{(p-1)}$

& $\tilde{F}_{(3)} = F_{(3)} - C_{(0)} \wedge H_3$

$\tilde{F}_{(5)} = F_{(5)} - \frac{1}{2} C_{(2)} \wedge H_3 + \frac{1}{2} B_{(2)} \wedge F_{(3)}$

& we must impose that $\tilde{F}_{(5)} = \star \tilde{F}_{(5)}$ Hodge dual in 10-d

(cannot easily be imposed as a requirement into the action)

<u>Het</u>

Het

$$S_{RR} = \frac{\alpha'}{8\kappa^2} \int d^{10}X \sqrt{-G}\, \mathrm{Tr} |F_{(2)}|^2$$

$$F_{(2)} = dA_{(1)} + A_{(1)} \wedge A_{(1)} \rightsquigarrow \text{non-Abelian field strength}$$

Heterotic string only makes sense (anomaly free) for gauge group

$$SO(32) \quad \text{or} \quad E_8 \times E_8$$

(iii) Supersymmetry

These actions have the important property that together w/ S_{fermi}, they are invariant under local supersymmetry in spacetime relating bosons & fermions.

$$\text{Type IIA/IIB} \iff N=2 \ \text{SUSY}$$

$$\text{Heterotic} \iff N=1 \ \text{SUSY}$$

Such theories of GR + supersymmetry are called supergravity theories & the actions + field content are uniquely determined by supersymmetry.

Digression

$$G_{5-d} = \begin{pmatrix} (\gamma_{4-d}) & A \\ A & \underline{\underline{\phi}} \end{pmatrix}$$

$$\Downarrow$$

4-d theory: GR + Maxwell + $\underline{\underline{\text{Dilaton}}}$

IV - Simple solutions to low energy effective ...

IV - Simple solutions to low-energy effective actions

(i) Compactifications

Bosonic string theory requires $D=26$ but we only observe $D=4$! One way to make sense of this is to consider string compactifications:

$$M^{26} = \mathbb{R}^{1,3} \times \underbrace{M^{int}}_{\substack{\text{small \& compact} \\ \text{22-manifold}}}$$

small & compact 22-manifold $\Rightarrow$ "compactification"

Easiest class is $H=0$, $\phi = \text{const}$

$\Rightarrow R_{\mu\nu} = 0$ on M^{int}.

E.g. T^{22}

Consider LEEA:

$$S_{LEEA} = \frac{1}{2\kappa^2}\int d^{26}\sqrt{-G}\,R \sim \frac{Vol(M^{int})}{2\kappa^2}\int d^4x\sqrt{-G_{(4)}}\,R_{(4)}$$

Effective 4-d Newton constant

$$8\pi G_N^{(4-d)} = \frac{\kappa^2}{Vol(M^{int})}$$

$$\Rightarrow G_N^{(4-d)} = \frac{G_N^{(26-d)}}{Vol(M^{int})} \to Vol(M^{int}) \text{ weakens gravity!}$$