Lecture 20

Monday, 15 February 2021 09:05

Recap:

- Superstring massless sector & low-energy effective actions
  - → IIA, IIB & Heterotic theories
  - → common NS sector: g, B, ϕ
  - → "RR" sector for each is different
    - IIA odd p-forms
    - IIB even p-forms
    - Heterotic gauge fields ($E_8 \times E_8$ or $SO(32)$)

- Einstein vs string frame due to scalar coupling
  - → violates equivalence principle
  - → "moduli stabilisation"

- Compactification: $M^{26} = \mathbb{R}^{1,3} \times \underbrace{M^{int}}_{\text{compact e.g. } T^{22}}$

$$\rightarrow G_N^{(4d)} = \frac{G_N^{(10d)}}{\mathrm{Vol}(M_{int})}$$

(ii) String background

String has tension ⇒ causes curvature of G

String couples to B ⇒ charged under B

$\updownarrow$

sources B-field (c.f. electromag.)

Consider infinite static string stretched along X^1 direction

⇒ $G, B \& \phi$ are given by

$$ds^2 = f(r)^{-1}(-dt^2 + dX^2) + \sum^{25} dX^2$$

$$ds^2 = f(r)^{-1}\left(-dt^2 + dX_1^2\right) + \sum_{i=2}^{25} dX_i^2$$

$$B = \left(f(r)^{-1} - 1\right) dt \wedge dX_1$$

$$e^{2\phi} = f(r)^{-1}$$

constant number　　dimensional grounds

w\ $f(r) = 1 + \dfrac{g_s^2 N \ell_s^{22}}{r^{22}}$, $r = \sum_{i=2}^{25} X_i^2$

$f(r)$ is harmonic in transverse space

$$\nabla^2_{\mathbb{R}^{24}} f(r) = 0 \quad \text{except @ } r = 0$$

Let us now determine N. To do so, let's look at the B EoM. We want to solve for B w\ a source

$$S_{tot} = \underbrace{\frac{1}{g_s^2}\int_{\mathbb{R}^{26}} |H_{(3)}|^2}_{\text{LEEA}} + \underbrace{\int_{\mathbb{R}^2} B_{(2)}}_{\text{string action}}$$

$$= \frac{1}{g_s^2}\int_{\mathbb{R}^{26}} \left(H_{(3)} \wedge * H_{(3)} + g_s^2\, B_{(2)} \wedge \underbrace{\delta(w)}_{} \right)$$

δ-function w\ support on string worldsheet

$\Rightarrow$ EoM: $d * H_{(3)} \sim g_s^2\, \delta(w)$

Analogue of Gauss' law $d * F \sim \rho$

Analogue of Gauss' law $d{*}F \sim \rho$

Integrate over transverse space & use Stoke's Thm:

$$\Rightarrow \frac{1}{g_s^2}\int_{S^{22}} {*}H_{(3)} = 1$$

If we had N fundamental strings

$$\Rightarrow \frac{1}{g_s^2}\int_{S^{22}} {*}H_{(3)} = N$$

# of parallel fundamental strings!

$$\Rightarrow f(r) = 1 + \frac{g_s^2 N \ell_s^{22}}{r^{22}}$$

Knowing that strings are fundamental microscopic objects we must have $N \in \mathbb{Z}$.

These arguments are very important for AdS/CFT correspondence (c.f. Forini's course next semester).

V - Open String Sector

We saw that requiring conformal invariance when coupling to background fields gave rise to a low-energy effective action for the closed string excitations

$$G,\ B,\ \phi.$$

What about the open string excitations A_a, ϕ^I?

(i) D[illegible]Infeld action

(i) Born-Infeld action

Focus on A_a for now. Let's see how its background field couples to the open string.

Vertex operator

$$V \sim \int_{\partial\Sigma} d\tau\, \zeta_a\, \partial^\tau X^a\, e^{ip\cdot X}$$

boundary of worldsheet

↓

state-operator map for open string maps states to operators on boundary!

Now consider coherent state

$$Z \sim \int \mathcal{D}X\, \mathcal{D}g\; e^{-S_{poly}}\, e^{-V}$$

$$S_{poly} \to S_A = S_{poly} + \int_{\partial\Sigma} d\tau\, A_a(X) \frac{dX^a}{d\tau}$$

string endpoints are charged under A_a!

We can now compute β-function & find the effective action reproducing the condition for $\beta = 0$.

At 1-loop, this is given by the Born-Infeld action for Dp-brane for Dp-brane

$$S = -T_p \int_{\Sigma_{p+1}} d^{p+1}\xi \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab})}$$

tension

ξ^a are worldvolume coordinates of brane

$F_{ab} = \partial_a A_b - \partial_b A_a$

This is a highly non-linear theory! However, it's not as strange as it seems.
In the small field limit, $F_{ab} \ll \frac{1}{\alpha'}$, we can Taylor expand to get the Maxwell action

$$S = -T_p \int_{\Sigma_{p+1}} d^{p+1}\xi \left(1 + \frac{2\pi\alpha'}{4} F_{ab} F^{ab} + \dots\right)$$

suppressed by α'

As $F_{ab}\alpha'$ grows, the corrections resum into Born-Infeld action.

NB: There are further corrections involving higher derivatives:

$$\alpha' \partial F$$

coming from higher-loop β-functions which modify the Born-Infeld action.

(ii) Dirac-Born-Infeld action

(II) Dirac–Born–Infeld action

What about the scalar fields? These are fluctuations ϕ^I. The low-energy dynamics can be similarly deduced from a β-function calculation.

But it can also be argued for using Lorentz & reparameterisation symmetry. It should be given by Dirac action:

$$S = -T_p \int_{\Sigma_{p+1}} d^{p+1}\xi \sqrt{-\det \underbrace{X^*\eta}_{\gamma}}$$

w\ $X^\mu(\xi^a)$ the fluctuations of the D-brane.

The dynamics of scalars & vectors combines into the Dirac–Born–Infeld action:

$$S_{DBI} = -T_p \int d^{p+1}\xi \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab})}$$

w\ $\gamma_{ab} = (X^*\eta)_{ab} = \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$

In this action we have D scalar fields X^μ but we should only have $D-p-1$ physical excitations corresponding to transverse fluctuations.

Reparameterisation invariance means $p+1$ excitations are unphysical!

Can choose static gauge: $X^a = \xi^a$, $a = 0, \ldots, p$

$\Rightarrow$ Pull-back metric depends only on X^I (transverse):

$\Rightarrow$ Pull-back metric depends only on X^I (transverse):

$$\gamma_{ab} = \eta_{ab} + \frac{\partial X^I}{\partial \xi^a}\frac{\partial X^J}{\partial \xi^b}\delta_{IJ}$$

If we consider small field strengths & small gradients $\partial_a X^I$, we can expand DBI & focus on leading terms:

$$S = -(2\pi\alpha')^2 T_p \int d^{p+1}\xi \left(\frac{1}{4}F_{ab}F^{ab} + \frac{1}{2}\partial_a \Phi^I \partial^a \Phi^I + \ldots\right)$$

w/ $X^I = 2\pi\alpha' \Phi^I$ & we dropped a cosmological constant.

... are higher order in α'.

$\Rightarrow$ The low-energy effective action reduces to free Maxwell theory in $p+1$ dimensions coupled to $(D-p-1)$ free scalar fields.

(iii) Closed String Backgrounds

Consider now a Dp-brane in a curved background w/ G, B, ϕ from closed string sector turned on. The low-energy effective dynamics is still governed by a DBI-like action. Much of it can be argued for from symmetries:

$$S = -T_p \int d^{p+1}\xi \, e^{-\phi}\sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab} + B_{ab})}$$

where now

$$\gamma_{ab} = (X^* G)_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}.$$

This is the natural place for $G_{\mu\nu}$ to appear
$\rightarrow$ via the pull-back.

The dilaton appears as $e^{-\phi}$ because this effective action comes from tree-level of the open string
$\Rightarrow$ disc $\Rightarrow$ $e^{-\phi\chi}$ w\ $\chi = 1$ for disc!

The constant part $e^{-\phi_0} = g_s^{-1}$ modifies the tension of the Dp-brane:

$$T_p \Rightarrow T_p^{\text{effective}} = T_p \, g_s^{-1}$$

We see that the tension of the Dp-brane scales like g_s^{-1}

$\Rightarrow$ Dp-brane is a non-perturbative state:
it is ∞-ly heavy @ weak coupling
& it becomes light @ strong coupling

Finally,

$$B_{ab} = (X^* B)_{ab} = \partial_a X^\mu \partial_b X^\nu B_{\mu\nu}$$

has to appear in the combination

$$\mathcal{F}_{ab} = B_{ab} + 2\pi\alpha' F_{ab}$$

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since this is the only gauge-invariant combination

c.f. Sheet B, Qu.3

<u>NB</u>: also the DBI action contains the combination

$$\partial_a X^\mu \partial_b X^\nu (G_{\mu\nu} + B_{\mu\nu})$$

which is to be expected since this is the natural combination from the vertex operators.

(iv) Non-Abelian action

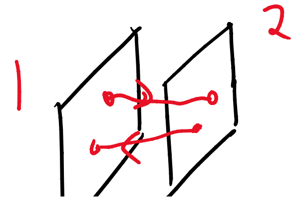
Everything so far was for single Dp-brane.
If we have N Dp-branes we get N^2 gauge fields

$$(A_a)^m{}_n \quad m,n = 1,\dots,N$$

By e.g. computing the scattering of these fields, we can see that we get a $U(N)$ gauge theory.
An alternative argument comes from realising that the

$$(A_a)^m{}_n$$

are charged under the various $U(1)$ subgroups of the individual D-branes.
E.g. for $N=2$



giving $(A_a)^1{}_2$ & $(A_a)^2{}_1$ which are charged under $U(1) \times U(1)$ $((A_a)^1{}_1, (A_a)^2{}_2)$ of the 1, 2 branes: $(+1,-1)$ & $(-1,+1)$

The only way to write down a consistent theory of these charged massless fields is via $U(2)$ Non-Abelian gauge theory.

$\Rightarrow$ N coincident D-branes $\Rightarrow$ $U(N)$ gauge theory

$\Rightarrow$ N^2 gauge fields

$\Rightarrow$ N^2 scalar fields which live in adjoint of $U(N)$

It is unclear what the non-abelian generalisation of DBI should look like (if it even exists).

E.g.

$$[D_a, D_b] F_{cd} = i [F_{ab}, F_{cd}]$$

for non-abelian gauge fields

$\Rightarrow$ Cannot clearly distinguish between DF expansion & const. F expansion.

Instead, restrict weak field limit $F \ll \alpha'$ & $DF \ll \alpha'$

$\Rightarrow$ low-energy action is given by YM coupled to scalars as:

$$S = -(2\pi\alpha')^2 T_p \int d^{p+1}\xi \, \mathrm{Tr}\left(\frac{1}{4} F_{ab}F^{ab} + \frac{1}{2} D_a\phi^I D^a \phi^I - \frac{1}{4}\sum_{I\neq J}[\phi^I, \phi^J]^2\right)$$

From this we can read off the YM coupling constant

$$\rightarrow \quad \frac{1}{g_{YM}^2} \mathrm{Tr}\, \frac{1}{4} F_{ab}F^{ab} \quad \text{in YM}$$

Using the precise relation between T_p & ℓ_s (c.f. Polchinski) we can show

$$g_{YM}^2 \sim \ell_s^{p-3} \, g_s$$